Effects of Immigration on Measuring Cohort Fertility

Felix Roessger
Max Planck Institute for Demographic Research, Rostock (Germany)
E-Mail: roessger@demogr.mpg.de

Extended abstract for submission to PAA 2012
(Sessions: 1001 *Formal Demography*
or: 110 *Advances in Measures and Models of Fertility and Sexual Behavior*)
September 23, 2011

Abstract

In this paper we examine the effects of immigration on the cohort total fertility rate (CTFR) as conventional measure for completed cohort fertility. As almost all conventional demographic measures it was designed for "closed" populations and can be consequently biased, if it is applied to an "open" population. We show that this bias arises from the fact that the immigrant population is not included in the population under exposure properly. Consequently, we propose a measure adapted for immigration, which treats migrants as if they always had been in the considered population. However, this new measure is restricted due to its high requirements on detailed fertility data. For this reason, a further iterative procedure is examined, which models the relevant process under certain assumptions but with less data requirements.

Introduction

Conventional demographic measures, such as life expectancy at birth or the total fertility rate, are usually designed for "closed" populations. Preston and Coale (1982) showed already 30 years ago formally that these measures can be adapted to "open" populations by including age-specific migration rates as well. However, applications or advancement of such measures are hard to find in the demographic literature.

Relevant research questions from a methodological point of view in this context can be thought of in two directions:

(1) How can conventional measures be misleading if they are applied to "open" populations?

(2) How can conventional measures be misleading if they are applied to migrant populations?

The first question was addressed, for instance, with regard to the net reproduction rate. Through providing results for a "(net) net reproduction rate", that is accounting for outmigration, Espenshade (1982) showed that the conventional measure could suggest non-realistic estimates of replacement-level fertility if outmigration is excluded. Preston and Wang (2007) advanced this idea by suggesting a net reproduction rate that allows for net migration.

The second question was addressed by Toulemon (2004) and Parrado (2011), who exhibited the issue that fertility of immigrants is more closely linked to the timing of migration than to the age of the migrants. Consequently, just summing up age-specific fertility rates could yield misleading estimates for the total fertility rate (TFR) of different immigrant groups, since not the entire fertility history of them is included. Therefore, the authors suggested adapted measures that include the entire fertility history of immigrants, which lead to dramatically different estimates for the TFRs of immigrants in the case of France (Toulemon 2004) or for Hispanics in the United States (Parrado 2011).

This study deals more with the first research direction but is also related to the findings of Toulemon (2004) and Parrado (2011). We will investigate how the not included part of the fertility history of immigrants matters for a conventional measure of cohort fertility – the cohort total fertility rate (CTFR). Usually we think of the TFR as distorted and CTFR as undistorted. This study reveals that also the CTFR can be biased, namely when it is applied on "open" populations.

Towards an immigration adapted measure of cohort fertility

Let us call f(x) the age-specific fertility rates of the total population at age x and N(x) the total age-specific population itself. A superscript "n" or "i" labels the respective numbers for natives or immigrants and β is the age at which the reproductive period ends. The conventional CTFR can then be written as

(1) $$\mathrm{CTFR} = \int_0^{\beta} \frac{f(x) \cdot N(x)}{N(x)} \, dx \, .$$

If we consider now immigration with I(x) being the immigrants added to the cohort at age x with the survival function l(x), we can express how the conventional CTFR deals implicitly with immigrants and their fertility as follows[1]:

$$\text{(2)} \qquad CTFR = \int_0^{\beta} \frac{f^n(x) \cdot N^n(x) + f^i(x) \cdot \int_0^x I(a) \cdot \frac{l(x)}{l(a)} da}{N^n(x) + \int_0^x I(a) \cdot \frac{l(x)}{l(a)} da} dx \, .$$

We see that immigrant fertility is only considered for immigrants who are present in the analyzed population. At ages where the immigrants are not present, the native fertility schedule gets the full weight. However, if the fertility schedule of immigrants is different from the one of the natives, this could bias our measure. If for example immigrants have exactly the same fertility schedule as natives, but shifted by one year to older ages, we would obtain a CTFR-value that is higher than the CTFRs of natives and immigrants. The reason is that the used correspondence between events and exposure is broken, because immigrants, in the case of delayed fertility, do not contribute exposure before they have arrived. This becomes particularly evident when we look at a small synthetic example. Let us assume a population with only two ages where everybody gets a child. The natives have their child at the first age and the immigrants at age two. If the immigrants add to the cohort only at age two, we observe on the total population level an age specific fertility rate of one child per women at the first age and an age-specific fertility rate that is smaller than one (but bigger than zero) at the second age. This leads to a conventional CTFR that indicates that the whole population has more than one child per women.

Table 1 illustrates this for a population of ten natives and ten immigrants for which we would estimate a CTFR of 1.5 children per women although both involved population subgroups have only one child during their reproductive period.

Table 1: Cohort fertility statistics for a model population with two age groups

Age	Natives	Immigrants	Total Population	Births to Natives	Births to Immigrants	Total Births	$f^n(x)$	$f^i(x)$	f(x)
1	10	0	10	10	0	10	1.0	0.0	1.0
2	10	10	20	0	10	10	0.0	1.0	0.5
						CTFR	1.0	1.0	1.5

For a CTFR-measure that includes both parts of the fertility history of immigrants – the one before and the one after migration, we suggest treating immigrants within this calculation as if they had been always in the population but with their specific fertility rates:

[1] The latter "a" also denotes age, but is labeled differently to "x" to avoid confusion in the double integral.

(3) $$CTFR^* = \int_0^\beta \frac{f^n(x)\cdot N^n(x) + f^i(x)\cdot \int_0^\beta I(a)da \cdot l(x)}{N^n(x) + \int_0^\beta I(a)da\cdot l(x)} dx\,.$$

Within the synthetic example in Table 1 this corrected measure, called CTFR*, would yield values of 0.5 for the age-specific fertility rates at age one and age two. Consequently, the value of the CTFR would be estimated with 1.0 children per women, which is exactly the figure from both population subgroups.

Unfortunately, the detailed data required for Formula (3) is rarely available. Age-specific fertility rates are usually not provided subdivided for natives and foreign born. Hence, it appears useful to consider models of this process which can be applied under certain assumptions with less data requirements. One option will be demonstrated in the following example:

If immigration starts only at the second reproductive age, the observed age-specific fertility rate at the first reproductive age is equal to the one of the natives. If we then have reason to assume that the fertility schedules of natives and immigrants are similar, but postponed by a certain time-lag (δ) for the immigrants due to the event of the move, we could get the native fertility-schedule back with help of an iterative procedure[2] - with $f^n(0)$ as initial value for estimating $f^n(1)$, $f^n(1)$ as initial value for estimating $f^n(2)$ and so on. For this procedure we only need the populations of foreign born and natives for estimating the age-specific shares of immigrants w(x), but no subdivided fertility statistics. Only the observed age-specific fertility rates for the total population are still required:

(4) $$f^n(x) = \frac{f(x) - f^n(x-\delta)\cdot w(x)}{1 - w(x)}\,.$$

If the fertility schedules of natives and immigrants are identical but only shifted, then the CTFR for natives equals our corrected measure CTFR*:

(5) $$CTFR = CTFR^* = \int_0^\beta f^n(x)\,dx\,.$$

We can test this with our example from Table 1 and reduce our “known” information by the subdivided fertility statistics for natives and foreign born:

Table 2: Cohort fertility statistics for a model population with two age groups (reduced information)[3]

Age	Immigrants	Total Population	Total Births	w(x)	f(x)
1	0	10	10	0	1.0
2	10	20	10	0.5	0.5
			CTFR		1.5

[2] The detailed derivation is available in full paper.
[3] In addition to Table 1 we calculated the shares of immigrants here which is simple I(x)/N(x) in our example.

With help of Formula (4) we can estimate the age-specific fertility rate of natives at age 2 which yields the “true” value of zero and we know that the age-specific fertility rate of natives at age 1 is equal to the observed one at this age (i.e. 1.0). Our estimate for the CTFR of immigrants and consequently for the CTFR* would than be the correct value of 1.0 children per women.

Conclusion

We showed that the conventional measure of completed cohort fertility, the CTFR, can be misleading under the presence of immigration, but also that it is possible to correct it for this bias. The magnitude of this distortion depends, on the one hand, on the share of immigrants in a considered population and, on the other hand, on the differences in the fertility schedules of natives and immigrants. Consequently, we expect relevant differences especially for smaller countries or regions where immigration plays an important role such as Switzerland or Israel. In the case of Israel fertility data by place of birth is available - at least for the Jewish population (see Central Bureau of Statistics 1996). Thus, the Israeli context appears a good one to apply the suggested correction with real empirical data. But since the availability of data for this research direction is usually very limited, the search for models with less data-requirements and their testing appears as essential part within this research project and within further research on this topic in general.

References

Central Bureau of Statistics. 1996. Statistical Abstract - Fertility Rates by Age, Religion and Continent of Birth. [http://www.cbs.gov.il/archive/shnaton47/st03-12.gif]

Espenshade, T.J. 1982. "A (net) net reproduction rate?" *Intercon, Population Reference Bureau* 10(11/12):8-10.

Parrado, E. 2011. "How High is Hispanic/Mexican Fertility in the United States? Immigration and Tempo Considerations." Demography 48(3):1059-1080.

Preston, S.H.and A.J. Coale. 1982. "Age Structure, Growth, Attrition, and Accession: A New Synthesis." *Population Index* 48(2):217-259.

Preston, S.H.and H. Wang. 2007. "Intrinsic Growth Rates and Net Reproduction Rates in the Presence of Migration." *Population and Development Review* 33(4):357-666.

Toulemon, L. 2004. "Fertility among immigrant women: new data, a new approach." *Population & Societies*(No. 400).