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# Equation Chapter 1 Section 1

# Is Fertility Too Low? Capital, Transfers and Consumption

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# Abstract

Fertility is below replacement level in much of the world, often far below replacement, leading to actual or projected population aging. Many governments view fertility as too low and have policies to raise it, while some still find it too high and have policies to reduce it. A key concern is the dependency cost of the elderly, perhaps partially offset by declining dependency costs of children. However, higher fertility leads to capital dilution, reducing the productivity of labor. Here we draw on age profiles of consumption and labor income from National Transfer Accounts for 32 countries around the world to calculate the level of fertility that would maximize, in steady state, 1) the general support ratio; 2) the fiscal support ratio; 3) life time consumption in an economy with a given capital-labor ratio; 4 ) life time consumption when both fertility and the capital-labor ratio are optimized, yielding the "goldenest golden rule". In most cases, fertility that maximizes the fiscal support ratio is quite high, well above replacement. Fertility that maximizes the general support ratio is near replacement level. When the capital-labor ratio is maintained at 4.0, the fertility that maximizes life time consumption is lower, generally between 1.5 and 2 births per woman. In less than half the cases (almost all rich countries) a goldenest golden rule economy exists (optimizing both fertility and capital intensity), with fertility lower still, between 1.0 and 1.6 births per woman. These results suggest that countries should not be overly concerned about below replacement fertility, at least from the point of view of costly intergenerational transfers.


Key words: Fertility, National Transfer Accounts, population aging, macroeconomic, demographic transition, economic consequences, capital, human capital, economic growth, support ratio, fiscal, optimal fertility.

## Introduction

Around the world, fertility is falling or is already low. Forty two percent of the world's population lives in countries in which fertility is below replacement (United Nations, 2011). In China, more than 500 million people live in administrative units where fertility is 1.3 births per woman or below. In Europe, average fertility is 1.5 births per woman. In Taiwan, it is .9. With fertility this low, population aging will be rapid and deep, resulting in heavy dependency burdens for the working age population, and severe strains on public pensions and health care programs. At the same time, fertility is still high in many countries, resulting in high child dependency ratios that impose heavy costs on private families and on public education and health care for children.

In 2009, 56 countries worldwide viewed their fertility as too high, and 51 as too low (United Nations 2011). If some levels of fertility are too high and others too low, is there some level that would be just right, an "optimal" level of fertility? Higher fertility reduces the costs of supporting the elderly, but raises the costs of investing in children and raises the costs of maintaining a given level capital intensity in the economy. The optimal level of fertility, as Samuelson (1975, 1978) noted, balances these costs and benefits. But it is important to keep in mind that this formulation ignores both the direct satisfaction that parents get from childbearing and the environmental consequences of continuing population growth.

The economic life cycle includes long periods of costly dependency in childhood and old age. Higher fertility raises the population share of children, while lower fertility raises the share of the elderly. Children and the elderly are largely supported by income transferred from the working ages and, in old age, previously accumulated assets may fund retirement as well. The Standard Support Ratio conveniently summarizes the relative amount of labor income by age and the relative amount of consumption by age. The support ratio is the population-weighted sum of labor income by age, divided by the population weighted sum of consumption by age, based on fixed age profiles and changing population age distribution. Other things equal, the level of consumption will be proportional to this support ratio. Fertility alters the population age distribution, and therefore the support ratio. We will seek the level of fertility that would maximize the support ratio in the long run (in steady state), for a given level of mortality.

Other kinds of support ratio are also of interest. For example, in modern welfare states programs transfer major resource flows to both children and the elderly. Much of the concern about low fertility is worry about the cost and fiscal sustainability of these programs in the face of population aging. We will seek the level of fertility that would maximize the *fiscal support ratio*, that is, the ratio of population-weighted age specific tax payments to population-weighted age specific costs of benefits.

Since the public sector is only a fraction of the total economy, we might also be interested in similar calculations for the private, or family, sector. Particularly in poor Third World countries, the public sector is often very small, so the private support ratio is highly relevant. It is calculated similarly to the general and fiscal support ratios, but based on age schedules of private transfers made and received.

More generally, we consider an economy with both capital and labor. We will ask what level of fertility leads to the highest life time consumption if the capital-labor ratio is held fixed at a given level by varying the saving rate. We also will seek the level of fertility that leads to what Samuelson (1975) called the "goldenest golden rule", that is, the level of fertility and population growth rate that would yield the best of all the possible golden rule steady states, optimizing both on capital intensity and on population age distribution. We will find that for some countries such an economy exists (at least as a local optimum) and for others it does not.

Our analysis is limited in many ways: it looks at comparative steady states rather than dynamic trajectories (for dynamic simulations, see Mason and Lee, 2007); it ignores direct satisfactions from childbearing; it assumes that the empirical age profiles remain fixed over time and across fertility rates; we do not consider that lower fertility would likely go with higher investment in each child's human capital (Lee and Mason 2010 explores this aspect); we ignore the consequences of population size. From a policy perspective, it is very important to consider not only the possibility of influencing fertility levels, but also interventions that modify the age profiles themselves through investing more in public health care and education or raising the age of retirement, for example. Nonetheless, although our analysis is limited, we believe it leads to useful insights.

# Theory

The analysis is concerned with the long-run implications of fertility and the population growth rate for macroeconomic objectives including improving public finances or achieving a higher material standard of living. We begin with some general principles and use these to examine public and private transfer systems, standards of living in consumption-loan economies, and standards of living in more realistic economies in which output depends on capital. The analysis is used to obtain analytic results for the fertility rate or population growth rate that maximize fiscal conditions for governments and standards of living. We show that the fertility rate that maximizes consumption is lower than the fertility rate that is most favorable for public finances. The results presented here draw heavily on a very substantial literature on these topics as will be discussed.

## Population and equivalence scales

We consider only steady state populations in which the total fertility rate (TFR), age-specific survival schedule, l(x), population growth rate, n, and age structure do not vary over time. We compare different steady state outcomes that would be realized given different total fertility rates or population growth rates. In the empirical section, we also consider how changes in the survival schedule or life expectancy influence outcomes, but in this section we hold the survival schedule constant.

All of the results presented here can be derived by considering only values in the base year for which t=0. Unless otherwise indicated then, all variables refer to values in the base year. The size of the population, *per se,* has no effect implications for any outcomes, because we rely on standard economic models in which scale has no effect on per capita values. We set the number of births in the base year to B=1 without loss of generality. The total population in the base year is given by:

$$N = \int_0^{\omega} e^{-nx} l(x)\,dx \tag{1.1}$$

where $e^{-nx}l(x)$ is the population at each age. The exponential term captures differences in the size birth cohort to which members of each particular age belong. If population is growing, for example, older members of the population will belong to smaller birth cohorts than younger member. The survival term, l(x), is the portion of the birth cohort that survives to age x.

The mean age provides a summary measure of population age structure which proves to be very useful. This is defined as:

$$A = \int_0^{\omega} x e^{-nx} l(x)\,dx \Big/ \int_0^{\omega} e^{-nx} l(x)\,dx. \tag{1.2}$$

Changes in age structure have important economic effects because ability, needs, and behavior vary with age. This age variation is captured using an empirically-based equivalent adult scale for each kind of economic variable. For example, the equivalent adult scale for labor income varies by age reflecting age variation in labor force participation, unemployment rates, hours worked, and wages or labor productivity. The equivalent adult scale for consumption varies with biological needs, tastes, decisions about spending on children, and many other factors. In all cases we take those who are 30-49 as our reference group and the equivalence value for those at any age are expressed relative to the average for that group. For the labor income equivalence scale, for example, a value of 0.5 for twenty-five-year-olds would indicate that they earn 50% on average of those who are 30-49 years old. The equivalence scales are based on cross-sectional estimates of the age profile of interest. In all analysis the equivalence scale is assumed to remain constant.

Equivalence scales are combined with population data to capture how changes in age structure affect the size of the population which serves some particular purpose. For example, weighting the population by the equivalent adult scale for labor income tells us the number of effective workers. In general, we will represent the effective population by $N_z$ and the equivalence scale $\psi_z$ where z the economic activity of interest. The steady state effective population for activity z is:

$$N_z = \int_0^{\omega} e^{-nx} l(x) \psi_z(x)\,dx. \tag{1.3}$$

The number of effective workers in the population is given by:

$$N_{y_l} = \int_0^{\omega} e^{-nx} l(x) \psi_{y_l}(x)\,dx. \tag{1.4}$$

The levels of economic variables are expressed relative to the effective population. Labor income per effective worker is denoted by $\bar{y}_l$ and consumption per effective consumer by $\bar{c}$. Aggregate values for the economy, represented by capital letters, are equal to the product of the effective population and the level of the variable. For example, $Y_l = \bar{y}_l N_{y_l}$. Because the equivalence age profile is fixed for economic variables, values at every age vary in direct proportion to changes in the level of that variable. Similarly, the maximum value at every age is realized when the value per effective population member is maximized.

The age distribution of any effective population can also be summarized by the average age. The average age for effective workers:

$$A_{y_l} = \int_0^{\omega} x e^{-nx} l(x) \psi_{y_l}(x)\,dx \Big/ \int_0^{\omega} e^{-nx} l(x) \psi_{y_l}(x)\,dx. \tag{1.5}$$

Many of the results presented below depend on how a change in the population growth rate affects the size of the effective population (relative to the size of the birth cohort). A well-known and useful steady state property is:

$$\frac{\partial \ln N_z}{\partial n} = -A_z \quad (1.6)$$

For an activity is concentrated late in life for example, an increase in the population growth rate will lead to a substantial decline in the effective population relative to the size of the current birth cohort. In an extreme case of an activity concentrated at age zero, an increase in population will have no effect on the size of the effective population relative to the birth cohort.

In the analysis presented below, we often present both the population growth rate and the fertility rate that maximize some particular objective. For a given survival schedule and any age distribution of childbearing, there is a one-to-one mapping between the population growth rate and the total fertility rate (TFR) in steady-state: $n \approx \ln\left(\frac{1}{2.05} l_\mu TFR\right) \Big/ \mu$, where 1/2.05 is the ratio of females to population at birth, $l_\mu$ is the proportion of births surviving to μ, the average age at which women give birth in the stable population, and $\mu$ is the mean age of childbearing. Typically, μ is around 30 years and this value is used for calculations presented below.

## Cross-sectional and longitudinal perspectives

Standard economics takes the individual longitudinal life cycle of consumption as fundamental. By assumption, individuals choose a consumption trajectory subject to a lifetime budget constraint. The consumption trajectory depends on tastes and interest rates. When interest rates rise, the price of current consumption relative to future consumption rises, and consumption trajectories rise more steeply over time.

The approach followed here and in much of our earlier work is to take the cross-sectional age distribution of consumption as fundamental reflecting altruistic links among the generations that are expressed through large systems of public and private transfers. Longitudinal profiles emerge from a series of shifts in the cross-sectional profiles due to productivity growth. In some respects our approach is in keeping with the Buffer-Stock model ((Deaton 1991; Carroll 1992). If higher productivity growth is matched by higher interest rates, as would be expected, and if consumption is sufficiently interest elastic, then these two approaches could generate qualitatively similar results in steady state.

Whether one takes the longitudinal or cross-sectional consumption profile as fundamental, all economic flows are governed by two constraints. The flow constraint applies to flows during any period and, simply put, says that income must be consumed, transferred, or saved. Using capital letters to represent aggregate flows, the aggregate flow constraint is that:

$$Y_l + Y_A + \mathrm{T} = C + S. \quad (1.7)$$

Where $Y_l$ is aggregate labor income, $Y_A$ is aggregate asset income, $\mathrm{T}$ is aggregate net transfers from the rest of the world, *C* is consumption, and *S* is aggregate saving. In a closed economy, total transfers, $\mathrm{T}$, will equal zero because any transfer outflow will be matched by a corresponding transfer inflow. In an open economy, total transfers will equal net transfers to the rest of the world.

The second constraint is the lifetime budget constraint that must hold for the newborn cohort. The newborn cohort enters the world with no assets. The present value of lifetime consumption must equal the present value of lifetime labor income plus lifetime net transfers. The longitudinal consumption profile of the cohort born in the base year is given by $e^{\lambda x}\psi_{y_l}(x)\bar{y}_l$. The variable $\bar{y}_l$ is the labor income of an equivalent worker in the base year. Given steady productivity growth, labor income of an equivalent worker will grow at rate $\lambda$. Labor income at each age will vary depending on the equivalence scale which captures labor productivity at each age relative to an equivalent worker. In similar fashion, consumption at each age over the lifetime of the newborn cohort is given by $e^{\lambda x}\psi_c(x)\bar{c}$ and net transfers at each age over the lifetime of the newborn cohort is given by $e^{\lambda x}\psi_\tau(x)\bar{\tau}$.

Designating the discount rate as *r,* the lifetime budget constraint for the newborn cohort is:

$$\int_0^\omega e^{(\lambda-r)x}l(x)\bar{y}_l\psi_{y_l}(x)dx+\int_0^\omega e^{(\lambda-r)x}l(x)\bar{\tau}\psi_\tau(x)dx=\int_0^\omega e^{(\lambda-r)x}l(x)\bar{c}\psi_c(x)dx. \qquad (1.8)$$

In order to simplify notation below, we define $PV_z$:

$$PV_z=\int_0^\omega e^{(\lambda-r)x}l(x)\psi_z(x)dx \qquad (1.9)$$

This is the lifetime present value for the new born cohort of effective labor force, or effective numbers of consumers, etc. Using this notation, the lifetime budget constraint can be rewritten as:

$$\bar{y}_l PV_{y_l}+\bar{\tau}PV_\tau=\bar{c}PV_c. \qquad (1.10)$$

An important feature of this model is the parallel between the effective population at a point in time and the present value of the effective population for the new born cohort. These two values, given in equation (1.3) and equation (1.9), are identical if the discount rate, r, is equal to the rate of growth of total income, $\lambda+n$.

## Transfer systems

Both saving and transfers are fundamental to reallocating resources across age. Given their enormous importance in all economies, however, transfers are of interest in their own right. Moreover, looking at transfer systems yields insights about the economy wide implications of changes in population age structure. Below we will introduce saving and capital into the picture, but in this section we will confine our attention to transfer systems: public transfers, private transfers, and the simple consumption-loan economy.

### The Public sector

Those who pay taxes tend to be concentrated in the working ages while public program beneficiaries tend to be concentrated at young and old ages. Thus, changes in age structure influence public finances by changing the concentration of the population in the taxpaying and beneficiary ages as measures by the number of effective taxpayers and the number of effective beneficiaries.

The issues to be addressed here are two. The first is whether a change in fertility or the corresponding population growth rate would improve public sector finances. The second related issue is what level of fertility (population growth) is most preferred from the perspective of public sector finances.

The public sector is characterized by taxes paid and benefits received at each age including all cash and in-kind transfers. The methods can be applied to sub-sectors, e.g., education, health, and pensions, but we concern ourselves here only with the public sector as a whole. We assume that equivalent adult scales for taxes paid and benefits received are given and expressed relative to the average tax payment or benefit of persons 30-49. The number of effective tax payers is given by $N_{\tau_g^-}$ and the number of effective beneficiaries is given by $N_{\tau_g^+}$ as defined above in equation (1.3). The level of taxation is measured by taxes per effective taxpayer, $\overline{\tau}_g^-$, and the level of benefits by benefit per effective beneficiary is $\overline{\tau}_g^+$. Total taxes collected in the base year is equal to $N_{\tau_g^-}\overline{\tau}_g^-$, the effective number of taxpayers multiplied the tax per equivalent taxpayer. Total benefits paid is equal to $N_{\tau_g^+}\overline{\tau}_g^+$, the effective number of beneficiaries times the benefit level, benefit per effective beneficiary.

The effects of population age structure can be fully captured by the fiscal support ratio, *SRG,* defined as the ratio of the number of effective taxpayers per effective beneficiary:

$$SRG = \frac{N_{\tau_g^-}}{N_{\tau_g^+}}. \qquad (1.11)$$

The fiscal condition of the government is measured as the natural log of the ratio of revenues relative to expenditure $\ln N_{\tau_g^-}\tau_g^- \big/ N_{\tau_g^+}\tau_g^+$. Note that if *Surplus*=0, the budget is balanced. The relationship between age structure and the fiscal condition in steady state is:

$$Surplus(n) = \ln\left(\overline{\tau}_g^- \big/ \overline{\tau}_g^+\right) + \ln SRG(n) \qquad (1.12)$$

This simple relationship can be used to assess two possible responses to changes in age structure – changes in the fiscal status of the public sector and changes in the levels and benefits required to maintain a given fiscal status. Moving the tax-benefit ratio to the other side, taking the natural log of both side, and differentiating with respect to the population growth rate yields:

$$\frac{\partial \ln Surplus(n)}{\partial n} + \frac{\partial \ln\left(\overline{\tau}_g^+(n) \big/ \overline{\tau}_g^-(n)\right)}{\partial n} = \frac{\partial \ln SRG(n)}{\partial n}. \qquad (1.13)$$

The impact of the population growth rate on the support ratio depends on whether the effective number of tax payers declines by more or the effective number of beneficiaries declines by more (relative to the size of the birth cohort) as the population growth rate changes. As shown above in equation (1.6) an increase in population growth leads to a decline in an effective population equal to the mean age of the effective population. Noting that $\ln SRG = \ln N_{\tau_g^-} + \ln N_{\tau_g^+}$ the derivative of ln SRG with respect to n is equal to the mean age of effective beneficiaries less the mean age of effective taxpayers and.

$$\frac{\partial \ln Surplus(n)}{\partial n} + \frac{\partial \ln\left(\overline{\tau}_g^+(n)/\overline{\tau}_g^-(n)\right)}{\partial n} = A_{\tau_g^+} - A_{\tau_g^-}. \tag{1.14}$$

In a relatively old, rich society with generous pension and health care provisions, effective taxpayers would be younger than effective beneficiaries. An increase in population growth would shift the age distribution of the population towards taxpayers and away from beneficiaries, allowing taxes to be reduced, benefits to be increased, or the budget surplus to be increased (or the deficit to be reduced). A slower population growth rate and older population is beneficial when beneficiaries are younger than taxpayers, a condition common in countries which are very young and have large public school budgets.

The most favorable fiscal conditions exist when the fiscal support ratio reaches its maximum. The first order condition for $n^{\max}$ is:

$$\frac{\partial \ln SRG(n)}{\partial n} = A_{\tau_g^+} - A_{\tau_g^-} = 0. \tag{1.15}$$

The second order conditions are met, although we will not show them here, because beneficiaries are concentrated at young and old ages while taxpayers are concentrated in the middle ages of the age distribution.

If all intergenerational transfers were public transfers, the fertility rate and rate of population growth rate that maximized the fiscal support ratio would be the same as the fertility rate and population growth rate that maximized the support ratio. This is shown to be the case below. But in a more realistic representation of the world, familial transfers also play a very important role. When this is the case, the fertility rate that is best for public finances differs from that which maximizes the support ratio and, in the consumption-loan economy, per capita income and consumption.

## Private sector and familial transfers

Changes in population age structure subject families to budgetary pressures similar to those experienced in the public sector. The same broad principles apply and similar analytic results can be readily obtained for family transfer systems. The family support ratio is constructed relying on age profiles of private transfers received and provided in a base year.

$$SRF(t) = \frac{N_{\tau_f^-}}{N_{\tau_f^+}} \tag{1.16}$$

where $N_{\tau_f^-}$ and $N_{\tau_f^+}$ are the effective number of providers and recipients of familial transfers. The interpretation of the family support ratio is very similar to that of the public support ratio. For our purposes here, we assume however that net private transfers to the rest of the world are zero. In this case total private transfer inflows must equal total private transfer outflows. As a consequence, the level of outflows relative to the level of inflows must equal the support ratio, and there is no familial transfer surplus. Letting $\overline{\tau}_f^-$ and $\overline{\tau}_f^+$ be the family transfer outflows per effective provider and inflows per effective recipient, respectively, we have:

$$N_{\tau_g^+}\overline{\tau}_g^+ = N_{\tau_g^-}\overline{\tau}_g^-. \tag{1.17}$$

Or rearranging terms and taking the natural ln yields.

$$\ln \overline{\tau}_g^+ / \overline{\tau}_g^- = \ln SRF. \tag{1.18}$$

Differentiating with respect to the population growth rate yields:

$$\begin{aligned} \frac{\partial}{\partial n}\ln\left(\overline{\tau}_f^+(n)/\overline{\tau}_f^-(n)\right) &= \frac{\partial}{\partial n}\ln SRF(n) \\ &= A_{\tau_f^+} - A_{\tau_f^-}. \end{aligned} \tag{1.19}$$

As will be seen below, those receiving familial transfers are heavily concentrated at young ages while providers consist of parents and to some extent grandparents. Hence, an increase in the population growth rate invariably increases the number of effective recipients (children) relative to the number of effective providers (parents and grandparents). The per capita private transfers received by children declines relative to the per capita private transfer made by adults. Although a maximum value might be possible if familial transfers to the elderly were sufficiently large, we do not find any cases where this occurs. This point has been discussed and demonstrated in several studies (Lee 1994; Lee 2003).

## Consumption loan economy

In this section, we turn to a broader issue of how changes in population age structure effect standards of living when intergenerational transfers are pervasive. This is the issue that Samuelson's (1958) consumption loan model addressed relying on a model of the economy in which all lifecycle needs are met by transfers.[1] There is no capital and labor is the only source of income. There are no durable goods and, hence, no saving. Under these conditions, all of labor income in a given period is consumed in that period, as expressed in the cross-sectional budget constraint in the base year:

$$\overline{c}N_c = \overline{y}_l N_{y_l} \tag{1.20}$$

Total consumption, consumption per equivalent consumer times the number of equivalent consumers, must equal total labor income, labor income per equivalent worker times the number of equivalent workers. The level of labor income is exogenously determined while the level of consumption is endogenous and determined by the support ratio, the effective number of workers over the effective number of consumers. Rearranging terms and defining the support ratio as $SR = N_{y_l}/N_c$, we have

[1] Samuelson also considers reliance on credit and rejects this as a possible solution to the lifecycle problem. We do not consider credit further here.

$$\bar{c} = \bar{y}_l SR$$
$$\ln \bar{c} = \ln \bar{y}_l + \ln SR \tag{1.21}$$

Relying on equation (1.6) the effect of a change in the population growth rate on the level of consumption is given by:

$$\frac{\partial}{\partial n}\ln \bar{c} \ = \frac{\partial}{\partial n}\ln SR = A_C - A_{y_l} \tag{1.22}$$

a result first derived by Arthur and McNicol (1978). If effective consumers are older than effective producers, a younger population achieved through higher fertility and more rapid population growth is advantageous. If the average age of consumption is 2 years greater than the average age of labor income, for example, then annual population growth at 1% compared to 0% would raise lifetime consumption by .01 times 2, or by 2%. Note that the mean ages of consumption and labor income will change as population growth increases. The mean age of consumption will decline relative to the mean age of labor income because labor income is more densely concentrated around its mean age than is consumption.

The first order condition for consumption-maximizing population growth is that the mean ages of consumption and labor income are equal, $A_C = A_{y_l}$. The second order condition is met because the variance of the consumption profile is always greater than the variance in the labor income profile due to the periods of dependency at the beginning and end of life.

In the simple transfer economy there are no credit markets or interest rates, but the transfer system yields an implicit rate of return that satisfies the lifecycle budget constraint,

$$PV_{y_l} = PV_c \tag{1.23}$$

By comparing the lifecycle budget constraint to the cross-sectional budget constraint it is clear that that interest rate that satisfies the lifecycle constraint is $n+\lambda$, called the biological rate of interest by Samuelson. Aggregate consumption and the present value of lifetime consumption are identical. The population growth rate that maximizes per capita consumption also maximizes the present value of lifetime consumption.

The value of this analysis is that it shows the circumstances under which the support ratio is an accurate indicator of the effects of age structure on standards of living. In economies without capital and complete reliance on transfers to deal with lifecycle issues, an increase in the support ratio leads to an increase in the level of consumption. The maximum consumption is achieved when the maximum support ratio is realized. And that is the age structure at which the average age of the effective consumer is the same as the average age of the effective worker.

## Introducing capital

Once the role of capital in any economy is acknowledged, the support ratio no longer fully captures the effects of population growth and population age structure on consumption. This is so because labor income will decline unless new capital is created to maintain the productivity of each new worker. This represents an additional cost of population growth because production must be diverted from consumption to saving to create new capital. If saving is insufficient to maintain the productivity of the workforce, then labor income and consumption will decline.

This is one of the important insights from Solow (1956). In the simple version of the neo-classical growth model the saving rate is held constant and all of the adjustment to more rapid population growth comes through lower capital per worker and lower productivity or labor income. Other versions of the neo-classical model treat saving in different ways. If saving rates are dictated by lifecycle motives, population growth will affect saving rates in ways about which there is not a firm consensus (Tobin 1967; Mason 1987; Mason 1988; Deaton 1991; Carroll 1992; Attanasio, Banks et al. 1999; Bosworth and Chodorow-Reich 2007). But irrespective of the response of saving, additional population growth will lead to lower labor income or higher saving. In either case, consumption is depressed. In earlier work, we have considered the effects of population growth on consumption given the lifecycle saving model (Mason and Lee 2007; Lee and Mason 2010; Mason, Lee et al. 2010). But here we will rely on several simple, but appealing, treatments of saving rates.

Two important implications of incorporating saving and capital into the analysis is clear from the cross-sectional budget constraint as represented in equation (1.25). The left-hand-side of the budget constraint is total consumption equal to the level of consumption times the effective number of consumers. The right-hand-side is total income less total saving with net transfers to the rest of the world assumed to be zero. Total income is calculated as the product of income, including both labor income and asset income, per effective worker ( $\bar{y}$ ) and the number of effective workers. Total saving is calculated as the product of the saving rate, s, defined as saving as a share of total income, and total income.

$$\bar{c}N_c = \bar{y}N_{y_l} - s\bar{y}N_{y_l} \quad (1.24)$$

Income per effective worker is endogenous and increases with capital per effective worker. But an increase in capital per effective workers comes at a cost – higher saving and lower consumption. Rearranging the cross-sectional budget constraint and gathering like terms yields:

$$\bar{c} = (1-s)\bar{y}SR. \quad (1.25)$$

Consumption per effective consumer is equal to the product of 1 minus the saving rate, income per effective producer, and the support ratio.

The relationship between capital and saving are not explicit in equation (1.25), but follows from the well-known steady-state condition derived by Solow:

$$s\bar{y} = (n+\lambda+\delta)k \quad (1.26)$$

## Case I. Fixed capital-output ratio.

This case is motivated by the experience of developed countries during the past thirty years or more. Since 1980, the average capital-output ratio for 14 OECD countries (Australia, Austria, Belgium, Canada, Finland, France, Ireland, Italy, the Netherlands, Spain, Sweden, Switzerland, the UK, and the US) has been remarkably constant with an average value of very close to 3.0. Capital-output ratios have also been quite stable with a few exceptions for individual countries. There is considerable variation among countries, with a value of 2.5 for the US, 3.1 for France, and close to 4 for Japan (Backus, Henriksen et al. 2008). The US capital-output ratio has been constant since 1948 (Evans 2008) . Holding the capital-output ratio fixed implies that the saving rate must vary in response to changes in the population growth rate – rising when the population growth rate increases and declining when the population growth rate declines. An advantage of slower population growth and population aging is that a lower saving rate will maintain the population growth rate (Cutler, Poterba et al. 1990). Dividing both sides of equation (1.26) by income per effective consumer, we have:

$$\begin{aligned} s &= (n+\lambda+\delta)\,k/\bar{y} \\ \frac{\partial s}{\partial n} &= \frac{k}{\bar{y}}. \end{aligned} \tag{1.27}$$

An increase in the population growth rate by one percentage point must be matched by an increase in the saving rate by three percentage points to maintain the OECD average capital-output ratio of 3.0.

The impact of population growth on steady-state consumption per equivalent consumer is found by taking the derivative of the natural log of the cross-sectional budget constraint in equation (1.24) with respect to the population growth rate:

$$\frac{\partial}{\partial n}\ln\bar{c} = \frac{\partial}{\partial n}\ln(1-s) + \frac{\partial}{\partial n}\ln SR. \tag{1.28}$$

The right-hand-side does not include any change in capital per effective worker, k, because capital per effective worker is constant following from the assumption that the capital-output ratio is constant.[2] Hence, income per effective worker is unaffected by the change in population growth, only the portion income that must be saved in addition to the change in the support ratio.

The first term on the right-hand-side is equal to $-\partial s/\partial n/(1-s)$. Substituting from equation (1.27) and from equation (1.22) we have:

$$\begin{aligned} \frac{\partial}{\partial n}\ln\bar{c} &= -\frac{k}{(1-s)\bar{y}} + A_c - A_{y_l} \\ &= -K/C + A_c - A_{y_l}. \end{aligned} \tag{1.29}$$

An increase in the population growth rate has smaller effect – less positive or more negative – on consumption per equivalent adult than captured by the support ratio. It also follows that the consumption maximizing population growth rate is less than the support ratio maximizing population growth rate. The maximum is realized when:

[2] This holds for the Cobb-Douglas production function, for example.

$$A_c - A_{y_l} = K/C. \tag{1.30}$$

Note that K/C and the mean ages of consumption and labor income vary with the population growth rate. $K/C = K/Y/(1-s) = K/Y/\left(1-\left(n+\delta+\lambda\right)K/Y\right)$.

## Case II. Golden-rule growth

The golden rule case has been discussed relatively extensively in the literature (see (Willis 1988; Lee 1994)). In the golden rule case, the saving rate, capital-labor ratio, and capital-output ratio adjust to insure the maximum possible consumption per equivalent consumer. Arthur and McNicol (1978) shows that across golden rule paths

$$\frac{d\ln\bar{c}}{dn} = -\frac{K}{C} + A_c - A_{yl}. \tag{1.31}$$

This the same first order condition that holds in the fixed capital-output ratio case, but in this case the capital-output ratio adjusts to changes in the population growth rate and is generally much higher than observed capital-output ratios around the world. For the special case of a constant returns to scale Cobb-Douglas aggregate production function, with capital coefficient $\alpha$, depreciation rate δ, and rate of total factor productivity growth λ, the golden rule ratio (K/C)*, is:

$$\left(K/C\right)^{*} = \left[\alpha/\left(1-\alpha\right)\right]/\left(n+\delta+\lambda\right) \tag{1.32}$$

From Willis (1988), Lee (1994a and b), and Lee and Mason (2011, Chapter 2), we know that the RHS of the first line of **Error! Reference source not found.** equals aggregate transfer wealth per capita (the difference between total life cycle wealth and the portion of it that is held as capital). We can conclude that the first order condition for the optimal level of fertility or population growth rate is that aggregate transfer wealth be zero, which is equivalent to the condition that the aggregate demand for life cycle wealth is exactly met by holdings of capital.

This value of n also corresponds to what Samuelson (1975) called the goldenest golden rule. He proved:
**SERENDIPITY THEOREM. At the optimum growth rate g*, private lifetime saving will just support the most-golden golden-rule lifetime state.**

This theorem states that at the optimal population growth rate g* (or n* in our notation) private saving will just support the golden rule steady state, but this is the same as saying that no aggregate transfer wealth is necessary to achieve the golden rule steady state, given the optimal life cycle planning and saving of the representative individuals in each generation. His theorem follows from our observation that derivative of life time consumption with respect to the population growth rate just equals transfer wealth, which must therefore be zero at the optimum.

This golden rule case leads to valuable insights about the economic consequences of population aging. If fertility declines to low levels the support ratio will decline, as emphasized above, but capital deepening will also occur leading to higher wages and more capital income. We see that the fertility rate that maximizes the support ratio is higher than the fertility rate that maximizes lifetime consumption when there is capital.

The golden rule case is an attractive assumption because it leads to elegant and simple results, but is obviously a very strong assumption to make. In most or all actual economies the K/C ratio is well below the golden rule level for various reasons including public sector transfers to the elderly and rates of time preference that weight present consumption highly relative to future consumption, dampening saving rates. Typical values of the K/C ratio are around 4, while for a stationary population with n=0 the golden rule ratio would be around 7 (see **Error! Reference source not found.** with $\delta$=.05 and $\lambda$=.02, with the capital coefficient $\alpha$=.33).

There is no closed form solution for the consumption maximizing population growth rate. The first order conditions for all of consumption-maximizing population growth rates include endogenous variables, the mean ages of consumption and labor income, and numerical methods are used to solve for the consumption-maximizing population growth rate. Details are available from the authors.

## Data

To address these questions we draw on National Transfer Accounts (NTA), a new set of economic accounts which document economic flows to and from ages in a manner consistent with National Income and Product Accounts. Research teams in 37 countries on six continents are currently collaborating in the construction of NTA. The theoretical foundations of the accounts build on Lee (1994a, b) and some details of the accounts and some preliminary results are reported in Lee, Lee, and Mason (2008) and Mason, Lee, et al. (The most recent and comprehensive treatment is Lee and Mason (2011). We will use the flows of consumption, labor income, and public and private transfers by single years of age. The estimates and methods are reported in Lee and Mason (2011) and on the NTA website – www.ntaccounts.org. We also make extensive use of demographic data on population, fertility, and survival. We rely primarily on World Population Prospects 2010 (United Nations 2011) for these data.

NTA estimates are constructed using household surveys, administrative data from government agencies, and National Income and Product Account data. The accounts measure how much those at each age consume and produce through their labor. Consumption is a comprehensive measure that includes both public and private consumption. The age profiles of public education and health consumption are estimated separately while per capita consumption of other public goods and services is assumed to be identical for all ages (public pension transfers are income and not consumption). The age profiles of private health and education are also estimated separately. Other private consumption is estimated using household survey data and equivalence scales that rise from 0.4 for children who are 5 or younger to 1.0 for those who are twenty or older. The same equivalence scale is used in every country, but consumption by age varies because of a variety of factors that influence the consumption of households in which children live (age at childbearing, fertility differentials by income, and so on). Labor income is a comprehensive measure that includes earnings, benefits, and two thirds of the estimated value of labor of self-employed workers including unpaid family workers. The resulting profiles reflect age specific variation in labor force participation, hours worked, unemployment rates, and productivity. Results presented here rely on estimates constructed by research teams in 32 countries with wide national differences in demography, political system, social policy, and level of development.

In all contemporary countries we have studied children and the elderly consume substantially more than they produce through their labor. The difference can be made up in one of two ways – by relying on transfers or on assets. NTA estimates of public and private transfers have been constructed for some, but not all, of the 32 countries for which consumption and labor income profiles have been estimated. Public transfers received are estimated separately for education, health, and public pensions. Other public transfers are allocated to each age group in proportion to its size. Private transfers consist of both inter-household transfers including net transfers to the rest of the world (e.g. remittances received) and intra-household transfers.

These age profiles are so central to the analysis presented below that they are shown for the United States (Figure 1) and the Philippines (Figure 2). The upper panels of the figures show consumption and labor income age profiles expressed relative to the average per capita labor income of persons 30-49 years old. The most striking difference in the two profiles is that consumption is very flat at adult ages in the Philippines, but rises sharply with age in the United States. This is a common feature that distinguishes the higher from the lower income countries. A larger share of lifetime resources is being consumed at older ages in high income countries.

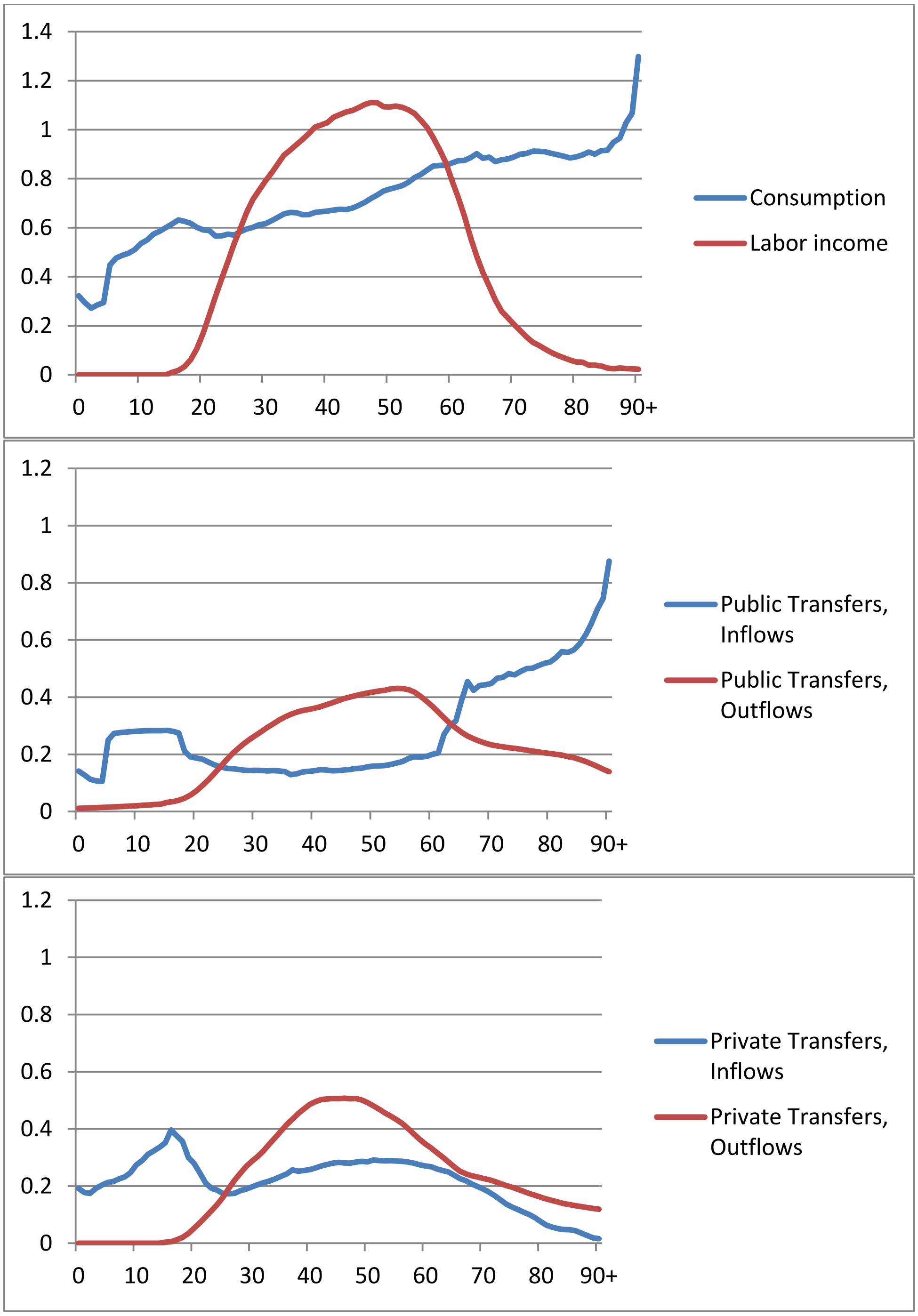


Figure 1. Age profiles of Consumption, labor income, and public and private transfers, United States, 2003. Note all profiles are expressed relative to the mean labor income of persons 30-49.

US profiles.xlsx

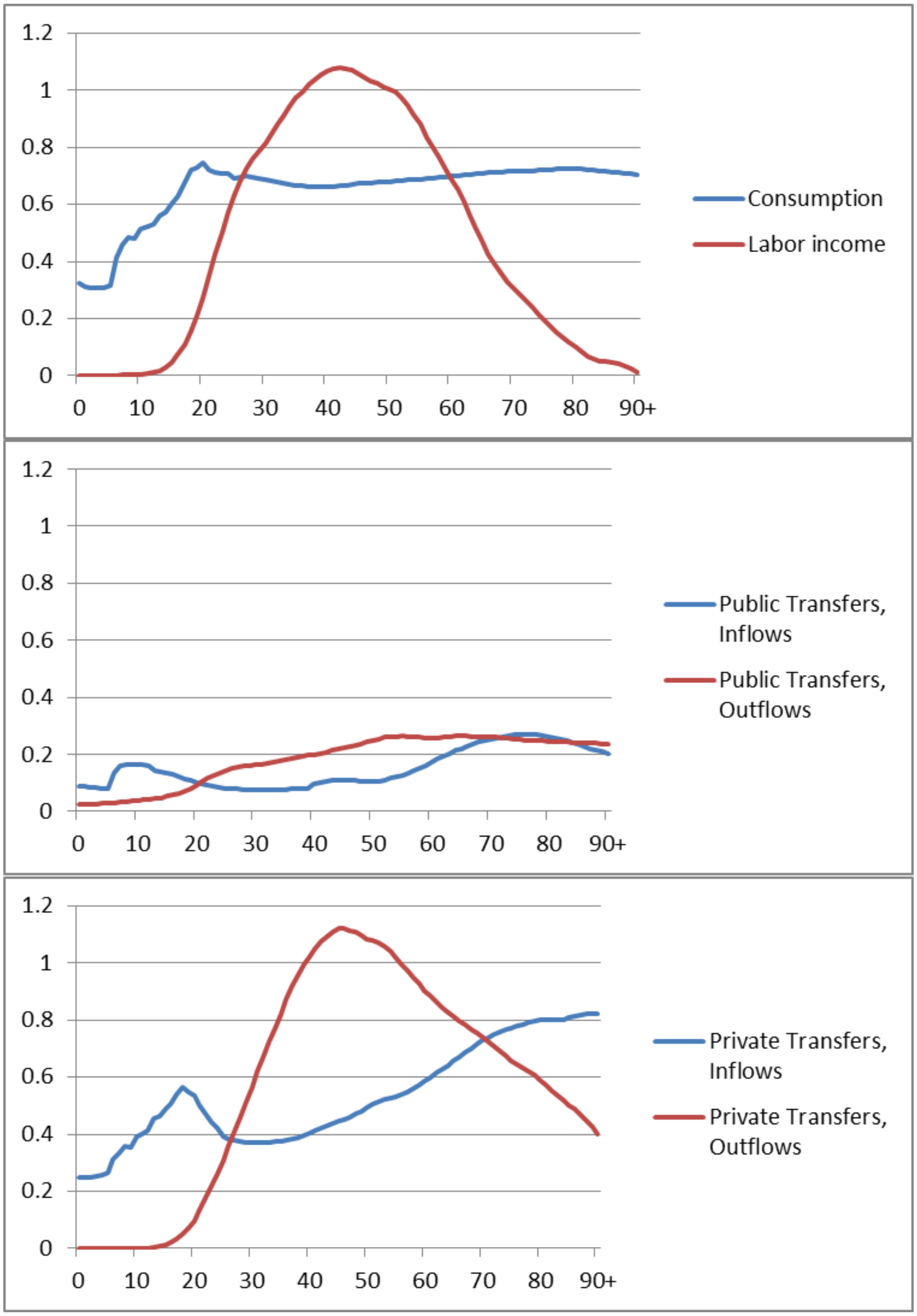


Figure 2. Age profiles of Consumption, labor income, and public and private transfers, Philippines, 1999. Note all profiles are expressed relative to the mean labor income of persons 30-49. phil profiles.xlsx

Public transfer inflows (benefits received) and outflows (taxes paid) are shown next in the figures. In both countries, the outflows are concentrated among adults and inflows to children are important in both countries. These flows primarily represent public schooling. The major difference between the US and the Philippines is the large public transfer inflows to the elderly in the US for health care and public pensions. This is the counterpart of the high public consumption among the elderly found in the US. Also note that tax payments drop off in old age in the US but not in the Philippines, so that the elderly in the Philippines pay their own way in the public sector, but have a huge deficit in the US.

The bottom panel shows private transfers. These are much larger in the Philippines than in the United States particularly at older ages. Below age 70 the elderly in the Philippines are giving more than they are receiving, however. At older ages the elderly are net recipients of transfers. In the US, net private transfers for the elderly are negative at all ages.

# Results

## Support ratios over the demographic transition

The emphasis in this paper is not on the dynamics of the support ratio over the demographic transition – a topic that we have discussed in a number of other papers. It is useful, nonetheless, to document briefly the trends in the support ratio and differences across countries. The values reported in Table 1 are based on NTA estimates of consumption and labor income profiles for each of 32 countries for which they are available and population projections from World Population Prospects 2010.

The first five columns of estimates document the dividend phase for each country which we define as the long period during which the support ratio is rising. For most developing countries in Latin America[3] and Asia, the dividend phase began in the late sixties or early seventies. The speed of the fertility transition is quite varied among these countries. The support ratio is currently near its peak in some countries like China, South Korea, and Thailand, but will occur somewhat later in Latin America and later still in the Philippines and India. Note that the African transition is expected to last for many years into the future. The total gain in the support ratio is substantial in the developing countries - over forty percent in Kenya, Nigeria, Philippines, Vietnam, South Korea, Taiwan, and Thailand and between 25 and 40% in most of the others.

Industrialized countries also experienced a dividend during this recent period, but it was qualitatively different because the bulk of the fertility decline had occurred much earlier in the late 19th and early 20th centuries. The more recent increase in the support ratio was a consequence of the entry of the baby boomers into the working ages. The duration of the divided phase was relatively short and the amplitude was relatively small. For Europe and the US, the average gain was only 12%. Japan -whose fertility transition was more standard and was not interrupted by a Baby Boom--falls in between the rich industrialized countries of the West and the developing world. Its dividend phase was relatively long-lasting and its support ratio increased by more than one quarter between 1950 and 1995.

In countries experiencing a slow or delayed decline in fertility, like the African NTA countries and Indonesia and the Philippines, the support ratio is not projected to decline much up to 2100. But for other countries the projected support ratios decline substantially from their maximum values by 2100 as

[3] Because some Latin American countries such as Uruguay and Chile had fertility transitions that began very early and then stalled, in these cases the dividend phase as assigned here refers to the more recent phase and abstracts from the earlier fertility decline.

their populations age.  Large declines of at least .23 are projected for low fertility East Asian economies, for Vietnam, for Costa Rica and Brazil, and for seven European NTA countries.  All of these countries share an important demographic feature – low or very low fertility at 2.0 or below. But the not all low fertility countries are in this category, and the shapes of their age profiles matter as well.

| Table 1. Support ratios at key points in the age transition. | | | | | | | |
|---|---|---|---|---|---|---|---|
| | Dividend phase | | | | | Value in 2100 | Decline from maximum to 2100 (%) |
| Country | Start year | Value | End year | Value | Gain (%) | | |
| Kenya | 1981 | 0.56 | 2092 | 0.82 | 45.1 | 0.81 | 0.2 |
| Nigeria | 2003 | 0.68 | 2100 | 0.96 | 41.3 | 0.96 | 0.0 |
| South Africa | 1973 | 0.78 | 2050 | 1.01 | 30.3 | 0.92 | 8.7 |
| ***Africa*** | ***1987*** | ***0.75*** | ***2084*** | ***1.00*** | ***35.3*** | ***0.98*** | ***2.3*** |
| China | 1972 | 0.66 | 2014 | 0.90 | 35.4 | 0.70 | 22.5 |
| Japan | 1950 | 0.67 | 1995 | 0.85 | 26.3 | 0.61 | 28.5 |
| South Korea | 1965 | 0.66 | 2006 | 0.94 | 41.7 | 0.70 | 25.1 |
| Taiwan | 1970 | 0.64 | 2010 | 0.93 | 45.5 | 0.65 | 29.5 |
| ***East Asia*** | **1964** | **0.66** | **2006** | **0.90** | **37.2** | **0.66** | **26.4** |
| Argentina | 1992 | 0.82 | 2030 | 0.91 | 10.4 | 0.78 | 14.1 |
| Brazil | 1967 | 0.66 | 2025 | 0.86 | 30.1 | 0.65 | 25.1 |
| Chile | 1969 | 0.76 | 2017 | 0.95 | 26.0 | 0.77 | 19.3 |
| Colombia | 1970 | 0.68 | 2023 | 0.92 | 36.3 | 0.79 | 15.0 |
| Costa Rica | 1970 | 0.69 | 2025 | 0.97 | 39.6 | 0.74 | 24.1 |
| Jamaica | 1975 | 0.71 | 2022 | 0.97 | 37.2 | 0.86 | 11.5 |
| Mexico | 1976 | 0.72 | 2033 | 0.98 | 36.3 | 0.85 | 14.1 |
| Peru | 1972 | 0.75 | 2033 | 0.96 | 27.0 | 0.80 | 16.6 |
| Uruguay | 1992 | 0.82 | 2025 | 0.88 | 6.3 | 0.77 | 12.0 |
| ***Latin America*** | **1976** | **0.74** | **2026** | **0.93** | **27.7** | **0.78** | **16.9** |
| India | 1973 | 0.81 | 2040 | 1.00 | 22.7 | 0.87 | 12.3 |
| Indonesia | 1977 | 0.76 | 2031 | 1.02 | 34.8 | 0.92 | 9.7 |
| Philippines | 1971 | 0.66 | 2056 | 0.93 | 40.4 | 0.88 | 5.8 |
| Thailand | 1972 | 0.70 | 2012 | 0.97 | 38.4 | 0.77 | 20.6 |
| Vietnam | 1973 | 0.70 | 2017 | 1.00 | 42.7 | 0.74 | 26.1 |
| ***S & SE Asia*** | **1973** | **0.73** | **2031** | **0.98** | **35.8** | **0.84** | **14.9** |
| Australia | 1967 | 0.88 | 1999 | 0.96 | 8.9 | 0.75 | 21.9 |
| Austria | 1972 | 0.78 | 1995 | 0.89 | 14.7 | 0.68 | 23.7 |
| Finland | 1965 | 0.78 | 1995 | 0.88 | 12.0 | 0.66 | 24.9 |
| Germany | 1972 | 0.76 | 1995 | 0.85 | 11.6 | 0.63 | 25.2 |
| Hungary | 1972 | 0.81 | 2010 | 0.86 | 7.0 | 0.70 | 19.1 |
| Slovenia | 1964 | 0.66 | 2006 | 0.76 | 15.5 | 0.55 | 27.6 |
| Spain | 1981 | 0.76 | 2011 | 0.90 | 19.1 | 0.66 | 26.9 |
| Sweden | 1950 | 0.85 | 1950 | 0.85 | 0.0 | 0.60 | 29.1 |
| U. K. | 1950 | 0.93 | 1950 | 0.93 | 0.0 | 0.71 | 23.9 |
| US | 1971 | 0.80 | 2005 | 0.91 | 13.6 | 0.73 | 20.0 |
| **Europe, USA, Australia** | **1966** | **0.80** | **1992** | **0.88** | **10.2** | **0.67** | **24.2** |
| Note: Support ratios are calculated using population projections from UN (2009) and NTA estimates of consumption and labor income by age. Regional values are simple averages of the reported country values. See text for more details about calculation method. | | | | | | | |

## Steady State Support Ratios

The remainder of this paper analyzes the long term effects of fertility, population growth rates and age structure through the prism of support ratios. We will start with the narrowest analysis which considers only fiscal support ratios, and then move to the standard support ratios. After that we will introduce capital and consider first the case of fixed capital-output ratios and then simultaneously vary capital intensity and fertility. The fiscal support ratio is an appealing place to begin because this bears on the long term sustainability of public transfer programs, a topic which has received a great deal of public attention.

## Fiscal support ratios

Fiscal support ratios can currently be constructed for 23 of the 32 countries of Table 1, but unfortunately none of the African countries yet have these available. Cross-national comparisons of the fiscal support ratio are not instructive as we have calculated them here, because the fiscal support ratio in the base year (year of the NTA estimate) is by definition equal to 1. Values for other years are expressed relative to the base year support ratio.

In Table 2, the first two columns report the base year TFR and the "intrinsic support ratio"– the support ratio in the stable population implied by the current fertility and mortality rates. The next two columns report the maximum fiscal support ratio and the corresponding TFR, while the final column reports the support ratio consistent with replacement fertility.

In South and Southeast Asia the intrinsic support ratio is slightly above 1.0. In other words, maintaining current demographic rates and public program structures would lead to improved public finances as compared with the situation today. The reason is that in these countries net public transfers to the elderly are relatively unimportant. Hence, population aging will reduce the population share of relatively costly children while the rise in the share of the elderly has little effect, since the elderly pay about as much in taxes as they receive in benefits.

In all other countries except Spain, the intrinsic support ratio is substantially less than one. The population aging implied by current fertility and mortality rates would adversely affect public finances relative to the base year. A typical value is about 0.8 meaning that population aging will require an decrease in per capita age specific benefits by 20% or an increase in taxes of 17% to maintain a balanced primary budget. There are no strong regional differences between East Asia, Latin America, and Europe/US. This may come as a surprise but there are several reasons. Public transfer programs to the elderly are more modest in East Asia than in Europe and in some Latin American countries, but East Asia's TFR is quite low and aging will be quite severe. Another factor is that European countries have already experienced more aging than Latin American and East Asian countries (other than Japan), so considerable population aging is already reflected in the base year situation.

| Table 2. Fiscal Support Ratio relative to base year values | | | | | |
|---|---|---|---|---|---|
| | | Intrinsic support ratio | Maximum support ratio | | Replacement support ratio |
| Country | Base year TFR | | TFR | Support ratio | |
| China | 1.6 | 0.82 | 2.6 | 0.86 | 0.86 |
| Japan | 1.3 | 0.73 | 2.7 | 0.84 | 0.83 |
| South Korea | 1.3 | 0.83 | 2.1 | 0.87 | 0.87 |
| Taiwan | 1.1 | 0.86 | 1.9 | 0.91 | 0.90 |
| **East Asia** | **1.3** | **0.81** | **2.3** | **0.87** | **0.86** |
| India | 2.7 | 1.02 | 2.0 | 1.02 | 1.02 |
| Indonesia | 2.2 | 1.08 | 0.9 | 1.23 | 1.08 |
| Philippines | 3.3 | 1.12 | 1.1 | 1.32 | 1.25 |
| Thailand | 1.6 | 1.16 | 0.8 | 1.27 | 1.06 |
| **S and SE Asia** | **2.5** | **1.09** | **1.2** | **1.21** | **1.10** |
| Argentina | 2.3 | 0.71 | 3.2 | 0.73 | 0.70 |
| Brazil | 1.9 | 0.71 | 5.5 | 0.93 | 0.74 |
| Chile | 1.9 | 0.77 | 3.6 | 0.87 | 0.79 |
| Costa Rica | 1.9 | 0.73 | 3.9 | 0.85 | 0.75 |
| Peru | 2.6 | 0.21 | 3.4 | 0.22 | 0.20 |
| Uruguay | 2.1 | 0.95 | 3.1 | 0.99 | 0.94 |
| **Latin America** | **2.1** | **0.67** | **3.8** | **0.76** | **0.69** |
| Austria | 1.4 | 0.72 | 4.0 | 0.95 | 0.85 |
| Finland | 1.8 | 0.86 | 2.9 | 0.91 | 0.88 |
| Germany | 1.4 | 0.77 | 3.3 | 0.91 | 0.87 |
| Hungary | 1.3 | 0.82 | 2.5 | 0.93 | 0.92 |
| Slovenia | 1.4 | 0.72 | 3.2 | 0.87 | 0.82 |
| Spain | 1.4 | 1.00 | 3.2 | 1.17 | 1.12 |
| Sweden | 1.9 | 0.90 | 3.5 | 0.98 | 0.92 |
| UK | 1.8 | 0.91 | 3.0 | 0.95 | 0.93 |
| USA | 2.1 | 0.89 | 2.1 | 0.89 | 0.89 |
| **Europe, USA** | **1.6** | **0.84** | **3.1** | **0.95** | **0.91** |

In the third and fourth columns, we find the level of fertility that would maximize the fiscal support ratio. The outcomes are driven by whether public transfers to the elderly for pensions, health care, and long term care outweigh public transfers to children for education and health care. In most countries transfers to the elderly dominate, so "optimal" fertility for the public sector is typically substantially higher than the current level, and far above replacement levels. This is true for rich industrial nations, Latin America, and East Asia. For Brazil a TFR of 5.5 would maximize the fiscal support ratio! However, the situation is very different for South and Southeast Asia, where public programs for the elderly are mostly very small and for children are relatively larger. In these countries it would be beneficial to have very low fertility to economize on transfers to children, and indeed we see that except for India, fertility close to 1.0 would be fiscally beneficial, and would lead to very high fiscal support ratios.

The final column of Table 4 reports the fiscal support ratio that corresponds to replacement fertility. In Brazil, Chile, and Costa Rica, among the Latin American countries, and Indonesia and Thailand, among the Southeast Asian countries, replacement fertility yields a substantial penalty relative to the maximum attainable support ratio. In Latin America, this is because replacement fertility produces a population that is too old, while in Southeast Asia, it is because replacement fertility produces a population that is too young.

The values presented in Table 2 are calculated using the base year survival rates for each country. To investigate the role of mortality differences, we recalculated the maximizing fertility level assuming that each country had the survival schedule of Japan. Doing so raises the maximizing level of fertility by .1 to .3 births, but does not change any of the patterns.

The bottom line is that outside of Southeast Asia, government budgets would benefit from a fairly high level of fertility. The median maximizing fertility in Table 2 is 3.0. But the public sector is only a fraction of the national economy, and outside of the rich nations that fraction is often quite small. We next take a broader view of the consequences of fertility for support systems by turning to the standard support ratio based on labor income and consumption.

## Steady state standard support ratios

Table 3 reports the long-term implications of three different fertility rates, with the implications expressed as the corresponding steady-state support ratio: first, the current TFR; second is the TFR which maximizes the support ratio; third is replacement level TFR.

The relationship between the intrinsic support ratio and the total fertility rate is hump-shaped. At high fertility rates, such as those found in Kenya and Nigeria, the support ratio is low because of the heavy dependency of children. The elderly in these countries (and particularly in Nigeria) tend to work far into old age and their consumption level is similar to that of younger adults. The average consumer is much younger in these countries than the average producer ($A_c<A_{yl}$) and, hence, a lower fertility rate and an older population will lead to a higher support ratio. Likewise, the intrinsic support ratio for very low fertility countries is quite low because the average consumer is much older than the average producer and, hence, a younger population – a higher fertility rate –would lead to a higher support ratio. The highest support ratios are clustered in countries with TFRs in the 2 to 3 range.

The maximum obtainable support ratio and the corresponding TFR are reported in the third and fourth columns of Table 2. In most countries the maximum support ratio is realized at a TFR near 2.0. In a few instances, like Nigeria and Indonesia, a very low TFR yields the highest attainable support ratio. A very old population is favorable in these countries because the elderly remain economically active and consumption is not high. The gap between consumption by the elderly and their labor income is much smaller (relative to the labor income of prime age adults) than elsewhere. For this reason, the economic burden of the elderly is much lower than in high income countries. In contrast, in Austria the TFR that maximizes the support ratio is 2.6. A young population is favorable there because labor income among the elderly is very low, while labor income is substantial for older teenagers due to the apprenticeship program there.

The high fertility NTA countries, Nigeria, Senegal and Kenya, have TFRs that are far above the level that would maximize the support ratio. The low fertility countries, Austria, Germany, Hungary, Japan, Taiwan and South Korea, have fertility rates that are well below the level that maximizes the support ratio. But how great would be the gain if these countries moved to the level of fertility that would maximize their Support Ratios?

Table 3. Steady-state support ratios, current survival rates.

| Country | Current TFR | Intrinsic support ratio | Maximum Support ratio | | Replacement support ratio |
|---|---|---|---|---|---|
| | | | TFR | Support ratio | |
| Kenya | 4.8 | 0.69 | 1.8 | 0.81 | 0.80 |
| Nigeria | 5.6 | 0.72 | 1.2 | 1.07 | 0.95 |
| South Africa | 2.6 | 0.91 | 1.5 | 0.97 | 0.92 |
| **Africa** | **4.5** | **0.85** | **1.4** | **1.04** | **0.98** |
| China | 1.6 | 0.77 | 2.1 | 0.78 | 0.78 |
| Japan | 1.3 | 0.64 | 2.3 | 0.70 | 0.70 |
| South Korea | 1.3 | 0.74 | 2.1 | 0.78 | 0.78 |
| Taiwan | 1.1 | 0.66 | 2.2 | 0.75 | 0.75 |
| East Asia | 1.3 | 0.70 | 2.2 | 0.75 | 0.75 |
| India | 2.7 | 0.93 | 1.9 | 0.96 | 0.95 |
| Indonesia | 2.2 | 0.98 | 1.3 | 1.03 | 0.98 |
| Philippines | 3.3 | 0.86 | 1.4 | 0.97 | 0.94 |
| Thailand | 1.6 | 0.84 | 2.0 | 0.85 | 0.84 |
| Vietnam | 1.9 | 0.83 | 2.5 | 0.85 | 0.84 |
| **South-East Asia** | **2.3** | **0.89** | **1.8** | **0.93** | **0.91** |
| Argentina | 2.2 | 0.86 | 2.0 | 0.87 | 0.87 |
| Brazil | 1.9 | 0.76 | 2.3 | 0.76 | 0.76 |
| Chile | 1.9 | 0.85 | 2.2 | 0.85 | 0.85 |
| Colombia | 2.4 | 0.86 | 2.1 | 0.87 | 0.87 |
| Costa Rica | 1.9 | 0.84 | 2.3 | 0.85 | 0.85 |
| Jamaica | 2.4 | 0.94 | 2.2 | 0.94 | 0.94 |
| Mexico | 2.4 | 0.92 | 2.0 | 0.93 | 0.93 |
| Peru | 2.6 | 0.88 | 2.2 | 0.89 | 0.89 |
| Uruguay | 2.1 | 0.85 | 1.9 | 0.85 | 0.85 |
| **Latin America** | **2.2** | **0.86** | **2.1** | **0.87** | **0.87** |
| Australia | 1.9 | 0.81 | 2.9 | 0.84 | 0.82 |
| Austria | 1.4 | 0.70 | 2.5 | 0.78 | 0.77 |
| Finland | 1.9 | 0.74 | 2.3 | 0.75 | 0.75 |
| Germany | 1.4 | 0.66 | 2.5 | 0.73 | 0.72 |
| Hungary | 1.3 | 0.76 | 2.0 | 0.79 | 0.79 |
| Slovenia | 1.4 | 0.59 | 2.2 | 0.63 | 0.63 |
| Spain | 1.4 | 0.71 | 2.2 | 0.75 | 0.75 |
| Sweden | 1.9 | 0.71 | 2.2 | 0.72 | 0.71 |
| UK | 1.8 | 0.78 | 2.6 | 0.81 | 0.80 |
| USA | 2.1 | 0.80 | 2.3 | 0.80 | 0.80 |
| **Europe, US, Australia** | **1.7** | **0.73** | **2.4** | **0.76** | **0.75** |

This question is answered by calculating the steady-state support ratio given the current age-specific fertility rates and age profiles for labor income and consumption for each country and the Japan survival rates. The percentage difference between that support ratio and the maximum attainable support ratio is plotted against the current TFR in Figure 3. As would be expected we see a U-shaped curve that is relatively flat in the 1.5 to 3 TFR range. For most countries in this range, the gain from moving to the maximum is just a small increase in the support ratio by about 2% or less. Brazil is an exception and could realize an increase in the support ratio of over 5% by moving to the SR-maximizing fertility rate. Outside this fertility range the gains are greater. Five of the low fertility countries, including Brazil, could increase their support ratio by 5-7%. Two low fertility countries, Germany and Austria, could gain around 12%. Among the high fertility countries, Nigeria's support ratio could rise by a considerable 30% while Kenya's gain would be about 10% by moving to the maximizing fertility level.

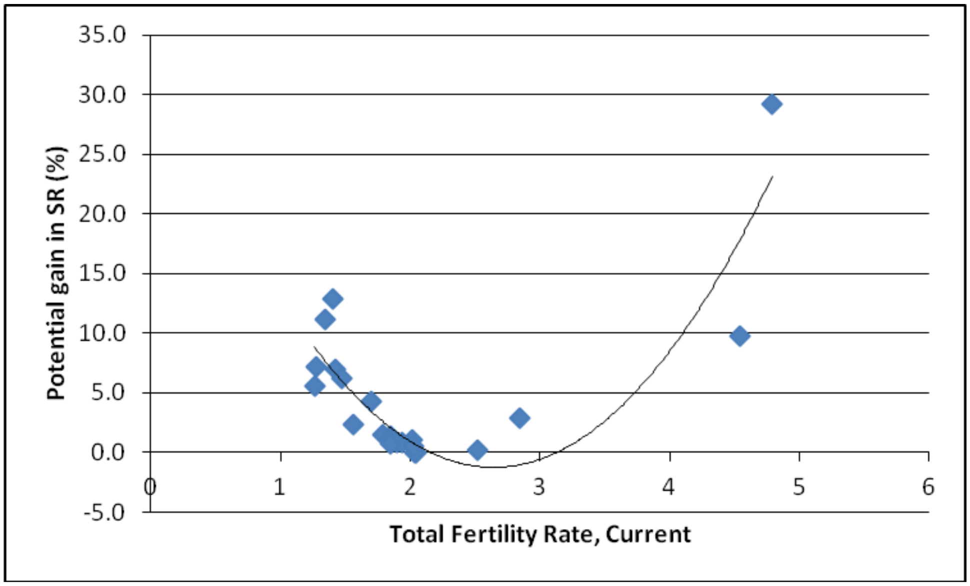


Figure 3. Percentage difference between the steady state support ratio given the current fertility rate and the maximum support ratio conditioned on the current age profiles of consumption and labor income and current Japanese survival rates.

These numbers may strike many as being surprisingly small. The reason is that these are steady-state results that do not reflect the large transitory effects of changes in age structure.

One of the most interesting aspects of these results is that the TFR that maximizes the support ratio is so close to replacement fertility. The final two columns in Table 2 compare the maximum support ratio to the replacement TFR support ratio. In almost every case the difference between the maximum support ratio and the replacement level support ratio seems almost trivial. The exceptions are mainly countries in which the lifecycle deficit of the elderly is quite small – Nigeria, Senegal and Indonesia.

This outcome occurs because the per capita economic lifecycles are roughly symmetric except in lower income countries. The periods of dependency at the beginning and end of life are of similar duration and magnitude so that over the lifecycle the mean ages of consumption and production are similar. Thus, in the aggregate the mean ages of consumers and producers are similar when the population age distribution is relatively uniform. It isn't obvious to us that there is some underlying mechanism that leads to this outcome and we do not observe it in more traditional settings, but it is worthy of further consideration.

## The support ratio and country differences in the lifecycle

The support ratios shown in Table 3 vary widely due to national differences in fertility, survival rates, and the economic life cycle. Now we consider to what extent these differences can be attributed to differences in the economic lifecycle versus differences in demography.

We isolate the influence of the economic lifecycle by holding the population age structure fixed while using the actual age profiles of consumption and labor income for each country, as shown in Figure 4. The population age distribution is calculated for replacement level fertility, and either the actual national mortality or recent survival rates for Japan, which had the highest life expectancy of any country in our sample.

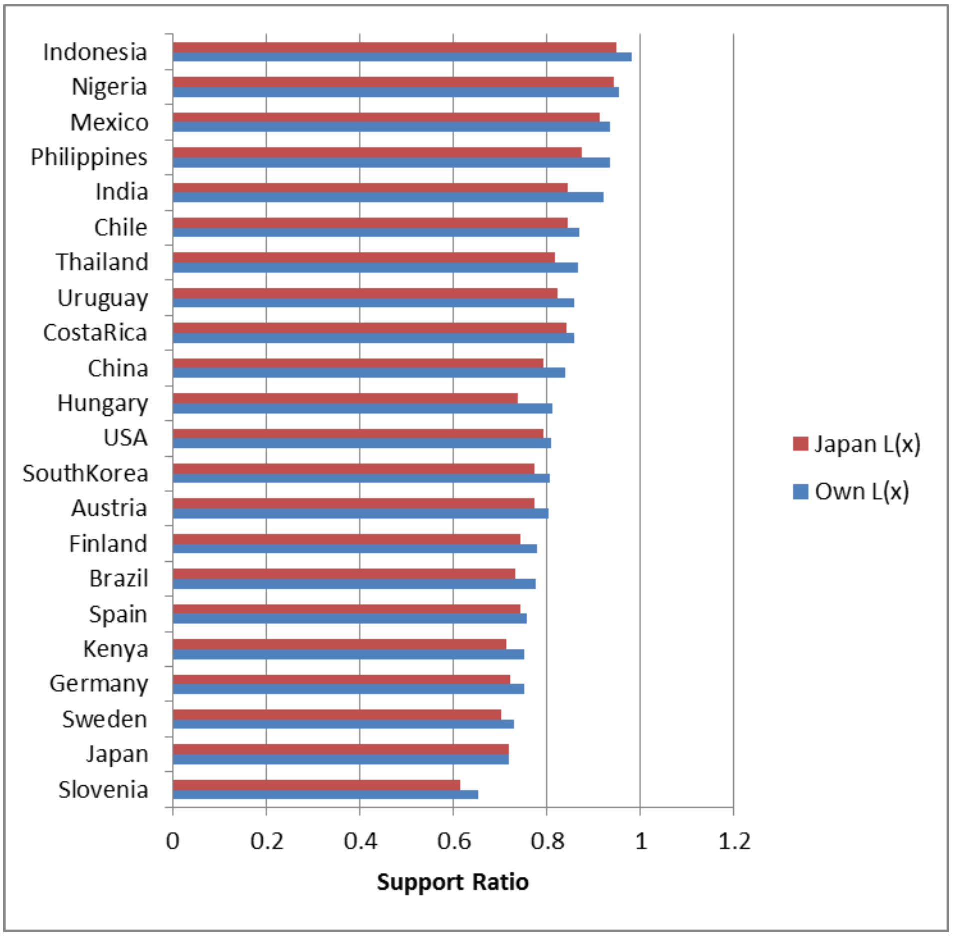


Figure 3. Percentage difference between the steady state support ratio given the current fertility rate and the maximum support ratio conditioned on the current age profiles of consumption and labor income and current Japanese survival rates.

These widely varying values show what the support ratio would be in each country given a steady-state population growth rate of zero. At one extreme, the replacement rate support ratio for Indonesia is 0.98 while in Slovenia, at the other extreme, the value is only 0.61. Other things equal, the economic lifecycle for Indonesia would accommodate consumption higher by more than 50% than the lifecycle in Slovenia if both had zero-growth population age distributions. If we consider the replacement level support ratios using survival rates for Japan, so that population age distributions are identical across countries, the basic pattern remains unchanged. The Indonesia support ratio would be somewhat lower at 0.94 and the Slovenia support ratio would be lower at 0.60. On average the support ratio is reduced for the 33 countries shown here from 0.837 to 0.798 given Japan's survival rates. This is a 5 percent decline in the support ratio due to the difference in life expectancy between Japan and the sample countries. Another indicator of the relatively modest impact of higher life expectancy on the support ratio is that the variance in the support ratio is reduced by less the 5% by controlling for differences in survival. The remainder is due to differences in the age profiles of consumption and labor income or the interaction of the profiles with survival rates.

The bottom line is that differences in the economic lifecycle can have an important effect on the support ratio. This finding is consistent with policy having a role in altering support ratios by, for example, raising the age at retirement or restructuring public transfers to children or the elderly.

This point is reinforced by calculations of the mean ages of consumption ($A_c$) and labor income ($A_{yl}$) for each country, abstracting from their demography by assuming that all have replacement level fertility and therefore zero population growth rates. This is done first for the actual current mortality of each country, and then again assuming that each country has the current mortality of Japan. In this way, the average ages result entirely from the age profiles of consumption and labor income. Table 4 shows cross-national variations in these mean ages, variations which suggest the scope for policies designed to change life cycle economic behavior. The mean age of earnings varies from 40 in Austria to 49 in Nigeria while the mean age of consumption varies from 42 in S. Korea to 47 in Brazil and the United States (see Table 4). The standard deviation of the mean age of consumption is 1.45 years and for earning is 1.92 years. Judging purely from the cross-sectional differences a one standard deviation increase in the mean age of earning would raise the support ratio by about 0.06.

Perhaps a more appropriate comparison is to look at countries with similar levels of development. Consider only the rich industrialized countries. In Japan and the US the mean age of earning exceeds 45, while in Germany, Spain, and Finland it is under 43 years, in Austria and South Korea it is about 42 years, and in Slovenia it is under 41 years. So raising the mean age of earning in these European economies to the level found in Japan and the US would yield very significant gains in the support ratio – about 0.1 in Austria and somewhat more in Slovenia, for example.

In the rich industrialized countries the mean age of consumption is highest in the US at over 47 years and lowest in Slovenia, Spain at 43 years and South Korea at 42 years. If the mean age of consumption in the US were to decline to the South Korean value, the support ratio would increase by about 0.15 a gain of about 20%. This is a crude approximation but gives a rough idea of the impact of alternative consumption and labor income profiles.

The far right column, giving support ratios for identical replacement level population age distributions, also shows striking differences. The US has the highest support ratio among all rich industrial nations, while Slovenia has the lowest, distantly followed by Sweden, Japan and Germany. Most but not all of the Third World countries have far higher support ratios, with the exception of Kenya and Brazil.

Table 4. Mean ages of consumption and labor income and support ratios in steady state populations with replacement fertility and own survival schedule or survival schedule for Japan.

| | Own survival schedule | | | | Japan survival schedule | | | |
|---|---|---|---|---|---|---|---|---|
| Country | AC | AYl | Ac-Ayl | Support ratio | AC | AYl | Ac-Ayl | Support ratio |
| Slovenia | 41.6 | 40.7 | 1.0 | 0.63 | 43.9 | 40.7 | 3.1 | 0.598 |
| Japan | 47.1 | 45.2 | 1.9 | 0.70 | 47.1 | 45.2 | 1.9 | 0.697 |
| Sweden | 45.4 | 44.0 | 1.4 | 0.71 | 47.5 | 44.0 | 3.5 | 0.68 |
| Germany | 45.7 | 42.6 | 3.0 | 0.72 | 47.4 | 42.7 | 4.7 | 0.69 |
| Spain | 43.3 | 42.8 | 0.5 | 0.75 | 44.4 | 42.9 | 1.6 | 0.727 |
| Finland | 44.2 | 42.7 | 1.5 | 0.75 | 46.4 | 42.9 | 3.5 | 0.718 |
| Taiwan | 41.1 | 41.0 | 0.1 | 0.75 | 43.1 | 41.2 | 1.9 | 0.723 |
| Brazil | 44.6 | 43.5 | 1.1 | 0.76 | 48.4 | 44.3 | 4.1 | 0.711 |
| Austria | 43.6 | 40.0 | 3.6 | 0.77 | 44.8 | 40.0 | 4.8 | 0.746 |
| S. Korea | 41.8 | 42.1 | -0.3 | 0.78 | 43.0 | 42.1 | 0.8 | 0.758 |
| China | 40.5 | 41.5 | -1.0 | 0.78 | 44.1 | 42.1 | 2.0 | 0.731 |
| Hungary | 41.7 | 42.6 | -0.9 | 0.79 | 46.2 | 43.1 | 3.1 | 0.718 |
| UK | 44.8 | 41.4 | 3.4 | 0.80 | 46.6 | 41.5 | 5.2 | 0.763 |
| Kenya | 37.6 | 41.0 | -3.4 | 0.80 | 45.8 | 44.8 | 1.0 | 0.763 |
| US | 46.8 | 45.3 | 1.5 | 0.80 | 49.1 | 45.7 | 3.4 | 0.766 |
| Australia | 45.3 | 40.9 | 4.4 | 0.82 | 46.0 | 40.9 | 5.1 | 0.807 |
| Vietnam | 41.0 | 39.0 | 2.0 | 0.84 | 43.5 | 39.2 | 4.4 | 0.801 |
| Thailand | 41.2 | 42.0 | -0.9 | 0.84 | 44.5 | 42.5 | 2.0 | 0.799 |
| Costa Rica | 45.3 | 43.8 | 1.5 | 0.85 | 46.8 | 44.0 | 2.8 | 0.821 |
| Uruguay | 42.7 | 44.1 | -1.5 | 0.85 | 45.4 | 44.5 | 0.9 | 0.806 |
| Chile | 44.4 | 43.4 | 1.0 | 0.85 | 46.0 | 43.7 | 2.4 | 0.824 |
| Argentina | 41.7 | 42.3 | -0.6 | 0.87 | 44.8 | 42.7 | 2.1 | 0.814 |
| Colombia | 43.4 | 43.6 | -0.2 | 0.87 | 46.7 | 44.3 | 2.4 | 0.823 |
| Peru | 43.2 | 43.1 | 0.1 | 0.89 | 46.6 | 43.7 | 2.8 | 0.836 |
| South Africa | 36.4 | 42.4 | -6.0 | 0.92 | 46.0 | 44.8 | 1.2 | 0.885 |
| Mexico | 43.0 | 43.8 | -0.8 | 0.93 | 45.1 | 44.3 | 0.8 | 0.897 |
| Philippines | 39.5 | 43.7 | -4.2 | 0.94 | 45.6 | 45.2 | 0.4 | 0.857 |
| Jamaica | 42.3 | 42.1 | 0.2 | 0.94 | 45.7 | 42.9 | 2.8 | 0.891 |
| Nigeria | 38.5 | 46.9 | -8.4 | 0.95 | 47.2 | 49.5 | -2.3 | 0.927 |
| India | 41.3 | 43.5 | -2.2 | 0.95 | 47.3 | 44.9 | 2.3 | 0.862 |
| Indonesia | 38.4 | 43.4 | -5.0 | 0.98 | 43.5 | 45.6 | -2.1 | 0.937 |
| Averages | 42.3 | 42.7 | -0.3 | 0.837 | 45.7 | 43.5 | 2.3 | 0.798 |

## Maximizing consumption when capital matters

As shown in the section on theory, maximizing the support ratio leads to the highest material standard of living only in very special and highly unrealistic circumstances – a consumption-loan or pure transfer economy in which capital does not exist. A more realistic treatment of the economy, however, requires the introduction of capital. We approach this in two different ways, as in the theory section. First, we will find the optimal growth rate and corresponding level of fertility when the capital labor ratio is fixed at 4.0, a value that is typical for most industrial nations, using **Error! Reference source not found.**. In our second approach, we assume that the capital stock adjusts to the golden rule value of the capital labor ratio as a function of n as in **Error! Reference source not found.**, and then search for the n which satisfies **Error! Reference source not found.**. In both cases we then find the population growth rate at which the derivative of life time consumption with respect to n is zero, if it exists, and check whether it corresponds to a minimum or maximum, and if so, whether it is local or global.

The results are shown in Table 5. First consider the case in which the K/L ratio is a constant equal to 4.0. The key point here is that the adverse economic effects of sub-replacement fertility on the support ratio are exaggerated if we ignore the effects of population growth on the capital stock or on the saving rate needed to prevent capital dilution. Our earlier analysis of the support ratio found that moderately positive population growth was desirable although moderate population decline involved little cost. However, once we add capital to the analysis, we find that modest rates of population decline, less than one percent per year, do not have adverse economic effects. Under some circumstances, a very old population and rapid population decline can be advantageous from a consumption maximizing perspective. These cases are of less interest because the countries involved are still relatively young, and their age profiles of consumption and labor income will likely change very substantially before they begin to experience substantial aging.

In every country the consumption maximizing population growth rate is negative. It is -4.9% per annum in Indonesia, but that is an extreme case. In the rich industrial nations, it is mostly near -1%, ranging from -.3% in Austria to -1.7% in Hungary. The consumption maximizing population growth rates in East Asia are somewhat lower ranging from -0.8% in Japan to -1.9% in South Korea.

Table 5. Summary measures given fertility rate that maximizes lifetime consumption, given age-profiles of consumption and labor income and current Japanese survival rates; K/C = 4.0 or at golden rule level, as indicated (see equations **Error! Reference source not found.**-**Error! Reference source not found.**.

| | | | K/L = 4.0 | | | K/L = Golden Rule | | |
|---|---|---|---|---|---|---|---|---|
| Country | AC | Ayl | TFR | Support ratio | Population growth rate (%) | TFR | Support ratio | Population growth rate (%) |
| Kenya | 49.3 | 43.7 | 1.3 | 0.74 | -0.025 | ---- | ---- | ---- |
| Nigeria | 62.2 | 56.1 | 0.6 | 1.00 | -0.057 | ---- | ---- | ---- |
| China | 50.3 | 44.7 | 1.3 | 0.77 | -0.019 | ---- | ---- | ---- |
| Japan | 52.3 | 46.4 | 1.6 | 0.68 | -0.008 | 1.3 | 0.65 | -1.5 |
| S Korea | 50.3 | 44.6 | 1.3 | 0.75 | -0.016 | ---- | ---- | ---- |
| Austria | 45.8 | 40.2 | 1.9 | 0.77 | -0.003 | 1.6 | 0.74 | -0.9 |
| Finland | 49.7 | 44.0 | 1.6 | 0.73 | -0.009 | 1.2 | 0.69 | -1.7 |
| Germany | 49.1 | 43.4 | 1.7 | 0.70 | -0.006 | 1.4 | 0.67 | -1.3 |
| Hungary | 50.4 | 44.8 | 1.3 | 0.77 | -0.017 | ---- | ---- | ---- |
| Slovenia | 47.2 | 41.7 | 1.6 | 0.62 | -0.009 | 1.3 | 0.58 | -1.7 |
| Spain | 50.0 | 44.3 | 1.5 | 0.72 | -0.011 | 1.1 | 0.67 | -2.1 |
| Sweden | 50.9 | 45.1 | 1.7 | 0.69 | -0.007 | 1.4 | 0.67 | -1.3 |
| US | 53.0 | 47.1 | 1.6 | 0.77 | -0.010 | 1.1 | 0.70 | -2.2 |
| Brazil | 51.1 | 45.3 | 1.6 | 0.73 | -0.011 | 1.0 | 0.64 | -2.8 |
| Chile | 51.4 | 45.6 | 1.5 | 0.82 | -0.013 | ---- | ---- | ---- |
| Costa Rica | 51.8 | 46.0 | 1.5 | 0.81 | -0.012 | ---- | ---- | ---- |
| Mexico | 53.7 | 47.9 | 1.2 | 0.89 | -0.020 | ---- | ---- | ---- |
| Uruguay | 52.6 | 46.8 | 1.3 | 0.82 | -0.018 | ---- | ---- | ---- |
| India | 54.1 | 48.3 | 1.2 | 0.90 | -0.027 | ---- | ---- | ---- |
| Indonesia | 60.8 | 54.7 | 0.5 | 0.93 | -0.049 | ---- | ---- | ---- |
| Philippines | 56.3 | 50.4 | 0.8 | 0.89 | -0.032 | ---- | ---- | ---- |
| Thailand | 50.9 | 45.2 | 1.1 | 0.83 | -0.023 | ---- | ---- | ---- |
| Taiwan | 48.3 | 42.7 | 1.5 | 0.73 | -0.012 | | | |

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Table 6. Summary measures given fertility rate that maximizes lifetime consumption, given age-profiles of consumption and labor income and current Japanese survival rates; K/C = 4.0 or at golden rule level, as indicated (see equations **Error! Reference source not found.**-**Error! Reference source not found.**.

| V | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| Country | Ac | Ayl | TFR | Support ratio | Population growth rate (%) | TFR | Support ratio | Population growth rate (%) |
| Kenya | | | 1.6 | 0.75 | -0.008 | 1.1 | 0.70 | -0.020 |
| Nigeria | | | 1.3 | 0.92 | -0.015 | 1.0 | 0.87 | -0.025 |
| Senegal | | | 1.2 | 1.12 | -0.017 | 0.7 | 1.02 | -0.035 |
| South Africa | | | 1.7 | 0.87 | -0.006 | 1.4 | 0.84 | -0.014 |
| China | | | 2.0 | 0.72 | -0.004 | 1.7 | 0.70 | -0.011 |
| Japan | | | 1.9 | 0.69 | -0.003 | 1.6 | 0.67 | -0.010 |
| South Korea | | | 1.7 | 0.75 | -0.007 | 1.4 | 0.72 | -0.013 |
| Taiwan | | | 1.9 | 0.72 | -0.004 | 1.6 | 0.70 | -0.009 |
| Australia | | | 2.3 | 0.82 | 0.003 | 1.9 | 0.79 | -0.004 |
| Austria | | | 2.2 | 0.75 | 0.002 | 1.9 | 0.73 | -0.004 |
| Finland | | | 2.0 | 0.72 | -0.001 | 1.7 | 0.70 | -0.006 |
| Germany | | | 2.2 | 0.70 | 0.002 | 1.8 | 0.67 | -0.004 |
| Hungary | | | 2.0 | 0.72 | -0.001 | 1.7 | 0.69 | -0.007 |
| Slovenia | | | 2.0 | 0.60 | -0.001 | 1.7 | 0.58 | -0.006 |
| Spain | | | 1.8 | 0.72 | -0.004 | 1.5 | 0.70 | -0.010 |
| Sweden | | | 2.1 | 0.68 | -0.001 | 1.8 | 0.66 | -0.006 |
| United Kingdom | | | 2.2 | 0.77 | 0.003 | 1.9 | 0.75 | -0.003 |
| US | | | 2.0 | 0.76 | -0.001 | 1.7 | 0.74 | -0.008 |
| Argentina | | | 1.9 | 0.81 | -0.004 | 1.6 | 0.78 | -0.010 |
| Brazil | | | 2.1 | 0.71 | 0.001 | 1.8 | 0.69 | -0.006 |
| Chile | | | 1.9 | 0.82 | -0.003 | 1.5 | 0.79 | -0.011 |
| Colombia | | | 1.9 | 0.82 | -0.003 | 1.5 | 0.78 | -0.011 |
| Costa | | | 1.9 | 0.81 | -0.003 | 1.6 | 0.78 | -0.010 |

| | | | | | | |
|---|---|---|---|---|---|---|
| Rica | | | | | | |
| Jamaica | 1.9 | 0.88 | -0.003 | 1.6 | 0.85 | -0.010 |
| Mexico | 1.7 | 0.88 | -0.008 | 1.3 | 0.84 | -0.016 |
| Peru | 1.9 | 0.83 | -0.002 | 1.6 | 0.80 | -0.010 |
| Uruguay | 1.8 | 0.80 | -0.006 | 1.4 | 0.77 | -0.013 |
| India | 1.9 | 0.85 | -0.004 | 1.6 | 0.82 | -0.012 |
| Indonesia | 1.3 | 0.92 | -0.018 | 0.9 | 0.86 | -0.030 |
| Philippines | 1.6 | 0.84 | -0.008 | 1.3 | 0.81 | -0.016 |
| Thailand | 1.9 | 0.79 | -0.004 | 1.6 | 0.76 | -0.010 |
| Vietnam | 2.1 | 0.81 | 0.001 | 1.8 | 0.78 | -0.005 |

(Statements about the goldenest golden rule case in this paragraph are not correct.) When we consider golden rule trajectories, rather than taking the K/C ratio as fixed, we find that low fertility and population decline are still more desirable. At the same time, fewer countries have an interior optimum. In the poorer Third World countries, the elderly typically receive little in transfers, while transfers to children are important. In these cases low population growth rates are advantageous, but these imply a high level of capital in golden rule. In these countries there is lower demand for capital to support old age consumption, since people work longer and consume less in old age. For this reason, the aggregate demand for capital is relatively low and does not reach the level needed for golden rule at any population growth rate. In Table 6 the goldenest golden rule coincides with fertility in the range of 1 to 1.6 births per woman, .3 to .5 births per woman less than in the case of K/C=4.0. The K/L ratios in goldenest golden rule are between 8 and 12 (not shown).

## Limitations of the analysis

Our analysis has been limited in important ways which we will now discuss.

Consumption and life cycle saving could be modeled in a neoclassical way, in which the life cycle trajectories vary endogenously with the interest rate and therefore with the capital intensity. But there are many reasons why we cannot expect the actual combined public and private consumption behavior to be determined in this way, such as in-kind public transfers for education and health care, parental determination of children's consumption, imperfections in credit markets, uncertainty, unknown time of death, and so on (Lee, Mason and Lee, 2008). Here we have instead used our NTA estimates of cross-sectional consumption profiles and labor income profiles as a basis for all these calculations, and taken them to be invariant in the face of different interest rates.

The first order conditions for optimality could be achieved either by varying fertility as in the calculations reported here, or by varying the age profiles of consumption and labor income, and therefore of public and private transfers. Most proposed policy responses to aging do not involve modifying the underlying demography through fertility or immigration. Instead they involve modifying the labor income age profile by inducing people to work longer or modifying the consumption age profile by cutting benefits or by inducing people to save more or pay higher taxes. Our purpose in calculating optimal fertility has not been to promote governmental fertility policies, but rather to provide a background for current concerns about low fertility.

Our analysis applies only to comparative steady states, and therefore to the long term consequences of fertility levels. But practical policy decisions confront transitory population age distributions and transitory economic situations. A more difficult analysis is required in the case of choosing optimal trajectories (Arthur-McNicoll, 1978; Cutler et al, 1991). In some cases a steady state with less than the golden rule amount of capital will be optimal (dynamically efficient, in the sense that no generation can be made better off without making another worse off), but the outcome will depend on the nature of intergenerational altruistic links.

We have abstracted from effects of population growth or size on the environment and on technological progress. We have also abstracted from the time costs of children. We have incorporated the costs of investment in children's human capital, since these costs are part of the consumption age profile. However, we have not considered the optimal investment in human capital, in parallel to the optimal investment in physical capital in the golden rule case.

We have largely ignored the direct utility that parents may derive from their children and their children's wellbeing, even though people mainly have children by choice, conscious of the private costs of childrearing. A policy that somehow altered fertility outcomes would have direct effects on parental utility from children in addition to the indirect age distributional effects at the macro-level on which we have here focused.

We have calculated results for countries at all levels of fertility and development. However, as high fertility countries move through the fertility transition, and as poor countries develop, their age profiles of consumption and labor income are likely to change considerably, with rising investment in human capital of children, and increased health care expenditures for the elderly. Our analysis is most relevant, therefore, for rich countries that already have low fertility.

## Discussion and Conclusions

There is a widespread perception by policy makers and the public that low fertility and the population aging to which it leads threatens our well-being and the sustainability of our institutions. This concern is focused mainly on the rising dependency burden of the elderly, which derives from the economic lifecycle in which the elderly consume much more than they produce.

United Nations data show that governments in many Third World countries view their fertility as too high, while more than 60% of developed nations view theirs as too low. Doubtless other factors besides age distribution influence these views. Nonetheless, it is interesting to compare the nations' fertility policies to an indicator of whether their actual fertility is above or below the optimal level as calculated to maximize the general support ratio. All five of those countries in which actual fertility exceeds the optimal level by more than .2 births have policies in place to reduce fertility. Of the eight countries with fertility within .2 births of the optimal level, seven, or all but one, have no policies to change fertility Six of the nine countries with actual fertility more than 2 below the optimum have policies to raise fertility. Thus the difference between actual and optimal fertility predicts fertility policy in 18 out of 22 cases.

Most policies to raise fertility involve transfers to children or their parents, and therefore they would change the fiscal profiles and perhaps the general economic age profiles that are the backbone of our analysis. Similarly, some policies to reduce fertility rely on financial incentives including fees and fines, and these would also alter the economic age profiles. However, family planning programs, which reduce fertility in poor countries by subsidizing contraceptives, disseminating information, and making it easier for couples to achieve the number of births that they already prefer, are more consistent with our analytic framework.

Concern about low fertility was the starting point of our analysis. It was addressed by calculating support ratios for many countries, which summarize the number of workers per consumer. Higher fertility would reduce the old age dependency ratio, but it would raise the youth dependency ratio, and the net effect is summarized by the support ratio. Very high or very low fertility both lead to lower support ratios and a heavier support burden. In between we find an optimal level of fertility that would maximize the level of support. We found that optimal fertility in most developed nations was fairly close to replacement level, although Austria and Germany had higher optima at 2.6 and 2.4 births per woman, levels more than 1.0 births above their current levels. For the higher fertility poor countries of the world, fertility near replacement or just below would maximize their support ratios given their current age profiles of consumption and labor income.

However, the most visible aspect of population aging is the pressure it puts on the budgets of public sector transfer programs for the elderly, notably pensions, health care and long term care, to some degree offset by the lower public expenditures required for children. Governments of many countries view their fertility as too low for this reason, and many have policies in place to raise it. We used the fiscal support ratio to address this more focused concern, and found that the level of fertility that maximizes the fiscal support ratio is typically well above replacement level. That is because the general support ratio includes the very large private expenses for raising children, but these are excluded from the public support ratio. Also, the net private transfers from the elderly to their children are excluded from the fiscal support ratio. If Europe had the mortality of Japan, fiscally optimal fertility would be 3.5, about two births more than the actual average for Europe in 2010.

In a more nuanced approach, analysis recognizes a tradeoff between the beneficial effect of higher fertility on the old age dependency burden, and its adverse effect on capital per worker and thereby on productivity. Now optimal fertility is lower, because with slower growth the saving necessary to maintain the capital-labor ratio is less. If saving rates adjust to maintain capital-labor ratios near their current levels, optimal fertility is between 1.5 and 2.0 in the rich countries, close to its actual levels.

(Statements about the goldenest golden rule case in this paragraph are not correct.) But with lower fertility increased capital intensity becomes desirable. Following Samuelson, we searched for simultaneously for the optimal level of fertility and the optimal intensity of capital. We find that for most of the rich industrial nations a double optimum of this sort exists, but it does not for Third World countries with the exception of Brazil. This goldenest golden rule level of fertility is lower still, between 1.0 and 1.6 births per woman, and the corresponding capital intensity is roughly twice current levels. So far, patterns of time preference and intergenerational altruism have not led any country to approach golden rule capital intensity, even at actual contemporary levels of fertility and population growth.

Our analysis included no direct parental satisfaction from raising children, and very important omission. Taking that into account would surely raise the optimal fertility levels we have found. But measuring fertility preferences is a difficult task. In most rich industrial nations, surveys indicate the couples would like to have around 2 children on average. Our analysis also did not include possible environmental costs and pressures on limited natural resources of larger populations, nor did it consider possible effects of population size and growth on innovation and technological progress. These kinds of effects are all difficult to quantify, but that does not mean that they are not important.

Our general conclusion is that rich industrial nations should not be too concerned about below replacement fertility, at least not on economic grounds. It is true that fertility above replacement level in these countries would ease fiscal pressures by reducing population aging, but that advantage evaporates if we take a broader view of the intergenerational transfers in the economy, and if we include capital in the calculations.

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