**PAA 2012 submission for presentation (or poster)**

**Title:** Age-specific Proportion of Shifted Deaths and the Tempo Effect in Period Mortality

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**Short abstract**

Tempo effects remain a controversial but also very interesting topic in mortality research. Although the existence and the origin of tempo effects are theoretically easy to prove, the methods and their need for tempo adjustment are still highly controversial. In this paper I present a method for deriving the age-specific proportions of shifted deaths. These proportions contain the period flow of deaths which were rescued due to the period mortality conditions. According to the logic of Bongaarts and Feeny, these saved deaths cause a tempo distortion in conventional period life expectancy. By using the age-specific proportions of shifted deaths, it is possible to reformulate the different methods for tempo-adjusted life expectancy without the assumption of a proportional change in period mortality. The empirical application presents that the reformulated tempo-adjusted life expectancy reflects more precisely the changes in period mortality conditions than the conventional life expectancy.

## 1. Introduction

The controversial topic of tempo effects in period mortality has been mainly deal with the question whether tempo effects distort period mortality indicator such as life expectancy (Bongaarts and Feeney 2002; Vaupel 2002; Bongaarts and Feeney 2008; Guillot 2008; Wachter 2008; Luy and Wegner 2009; Bongaarts and Feeney 2010). Only few publications show the unexpected phenomenon that the trend in period death rates fluctuates despite a continuous improvement in survival conditions of cohorts living during the analysed period (Horiuchi 2008; Luy and Wegner 2009; Feeney 2010). According to the logic of Bongaarts and Feeney (2002; 2008; 2008), these fluctuations are caused by tempo effects. They are accompanied by a temporary change in the number of deaths within a period in which mortality conditions have changed. This temporary change in the number of deaths is caused by those deaths which are shifted to higher or younger ages due to the improvement or worsening of survival conditions.

But the number of shifted deaths has to be considered in the period mortality analysis beside the observed period stock of deaths. Otherwise the derived period mortality indicators are distorted by tempo effects. Bongaarts and Feeney (2002; 2008; 2008) proposed several methods to adjust the period life expectancy for tempo effects. The main assumption of these methods is that the proportion of shifted deaths is constant over age. Under this proportionality assumption, all methods lead to the same tempo-adjusted life expectancy. A proportional change in mortality provides however a mortality scenario in which the tempo-adjusted indicators can also be interpreted as the average of last period life expectancies or as an estimation of the cohort life expectancy (Wilmoth 2005; Guillot 2008; Wachter 2008).

This paper presents in the first part a method for deriving the age-specific proportion of shifted deaths without any assumptions. This proportion is helpful to understand the consequence of changes in period mortality and the resulting postponmend of deaths. The empirical application of estimating the age-specific shift of deaths among Swedish men presents that the conventional life expectancy leads to an unexpected description of the real mortality conditions. In the second part, we apply the age-specific proportions of deaths to reformulate the existed methods for tempo-adjusted life expectancy. It can be shown that all methods are similar when we consider the age-specific amount of shifted deaths. Furthermore, the trend of the tempo-adjusted life expectancy presents a better reflection of the period mortality conditions.

## 2. Age-specific proportion of shifted deaths

The initial point for deriving the age-specific proportion of shifted deaths is the period intensity of mortality. Mortality intensity is based on age-specific death rates of the 2nd kind[1]. This kind of rate provides the ratio between the age-specific number of deaths and all individuals whether they have or have not experienced this event (Sardon 1994, p.132). According to Bongaarts and Feeney, the death rate 2nd kind *i(x,t)* is defined as[2]:

(1) $$i(x,t) = m(x,t) \cdot \frac{L_{t-x}(x)}{l(0)}$$

| | |
|---|---|
| $m(x,t)$ | Age-specific death rate 1st kind |
| $\frac{L_{t-x}(x)}{l(0)}$ | Birth standardised number of person-years at age $x$ during period t of the cohort *t-x* |

While this rate is often applied in fertility or nuptiality analysis, it is very uncommon to use it in mortality analysis. There are two main reasons. First, age-specific death rates of 1st kind are mainly applied in standard method of mortality analysis (e.g. life table method). This rate provides the ratio between the number of events and the number of person-years under mortality risk. In contrast to the rate of the 2nd kind, the denominator of the 1st kind rate includes only survived persons who are exposed to mortality risk. The other reason is the need of detailed data. The second term in Eq. 1 includes cohort data to estimate the birth standardised number of person-years. Therefore, the number of births, of deaths and detailed data of migration flows of at least 100 cohorts living during the analysed period are necessary to evaluate the birth standardised number of person years at each age during a period.

The sum of all age-specific incidence rates (death rates 2nd kind) provides the intensity of mortality. In the cohort perspective the intensity is always one because every individual have to die once in their life. The mortality intensity of a cohort (cohort mortality rate CMR(c)) is defined as:

(2) $$CMR(c) = \sum_{x=0}^{\omega} i(x,c) = 1$$

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[1] Also known as incidence rate, reduce rate or frequencies

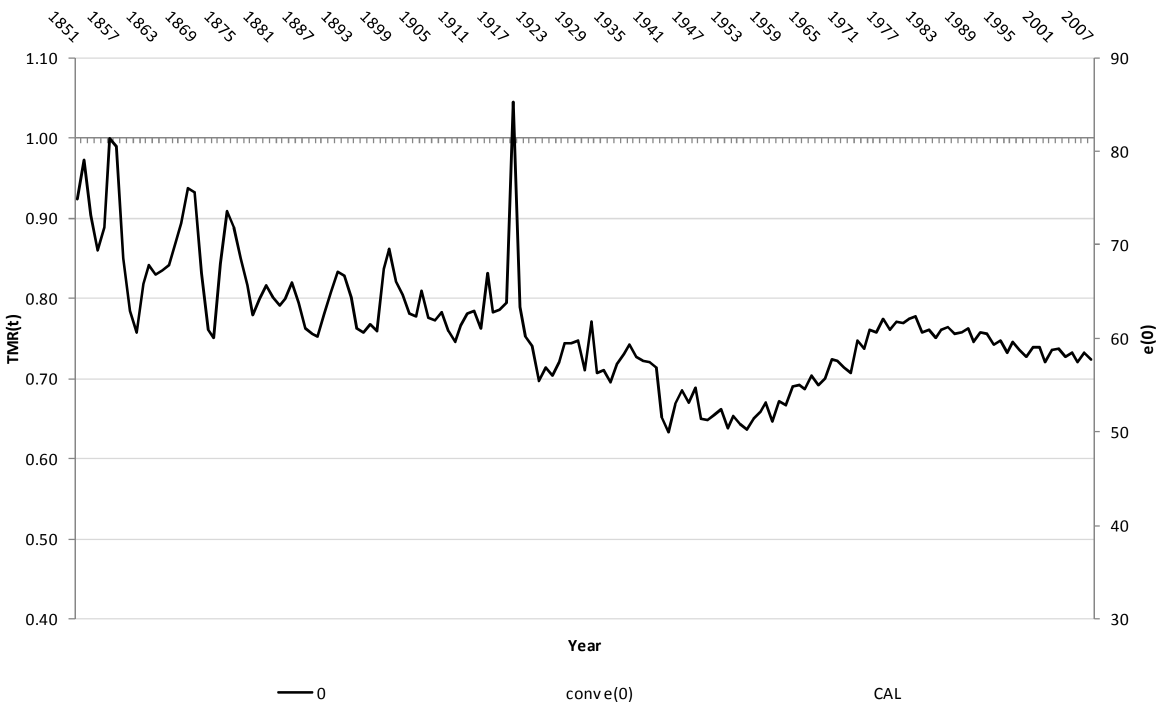
[2] All further formulas are expressed in discrete form. The discrete approach is mathematically undemanding but it is sufficient to derive the following relationships.

The period intensity (total mortality rate TMR(t)) is the sum of all age-specific incidence rates in a particular period t:

$$(3) \qquad TMR(t) = \sum_{x=0}^{\omega} i(x,t)$$

The interpretation of the TMR seems similar to other intensity measure like the total fertility rate. Thus, the TMR reflects the average number of events per newborn even if the incidence rates are constant over their life time. The expected value therefore must be one. But the TMR varies from one even if period mortality conditions have changed. The TMR is lower than one when survival conditions are improving or it is higher than one when mortality increases within a certain period. Figure 1 presents the trend in the TMR among Swedish men from 1851 to 2007.

**Figure 1 Trend of the total mortality rate (TMR) among Swedish men from 1851-2007**




Source: Human Mortality Database, own calculation

Except of two periods[3], the TMR is always lower from the expected value of one and, therefore, indicates a steady improvement of the period survival conditions. Since 1851 the TMR has decreased to the lowest value of 0.64 at 1953. This continuous reduction was mainly

[3] The first peak around 1856 was caused by starvation whilst the higher TMR at 1918 was caused by the Spanish flu epidemic.

caused by receding pandemics in the first half of the 20th century. After the 1950s the TMR increased to 0.76 during 1985 as a consequence of the increased smoking attributable mortality (Peto, Lopez et al. 2006). Since 1990 the TMR has again increased steadily to the current value of 0.72.

The divergence of the TMR from the expected value one requires rather a technical definition of the total mortality rate than a interpretation as a quantum indicator. Guillot (2008, p.132) proposed that the TMR presents “the proportion of cohort deaths that are occurring during the period t”. Based on this definition, a TMR lower from one indicates the observed stock of deaths from all cohorts during the period t. On the other side, the difference of the TMR from one can be characterised as the flow of deaths which has not occurred during the analysed period. The flow of deaths, therefore, may be characterised as the shift of deaths due to the improvement or worsening of period survival conditions.

The proportion of shifted deaths during a period S(t) may be formulated by the difference of the TMR from one:

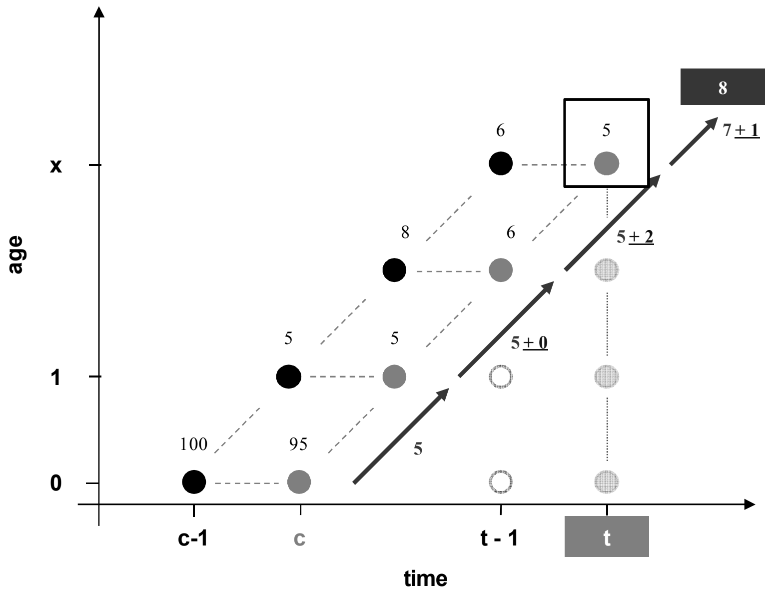
$$(4) \qquad 1 - TMR(t) = S(t)$$

Similar to the TMR, it is assumed that the total proportion of shifted deaths may be decomposed by the age-specific proportions of shifted deaths s(x,t):

$$(5) \qquad S(t) = \sum_{x=0}^{\omega} s(x,t)$$

A simple population model with constant births and no migration (Figure 2) will help to prove the assumption of Eq. 5 and to derive the age-specific proportion of shifted deaths. During the year t, five deaths (framed in Figure 2) occur at age x. In comparison to the previous year t-1 the number of deaths is reduced by one rescued death. But the number of shifted deaths at age x increases when considering the mortality history of both cohorts c and c-1 living during the analysed period (diagonal line). At age zero, the deaths of cohort c were reduced by five deaths compared to the previous cohort c-1. Thus, five infants could shift their deaths due to the reduction of the infant mortality. During the next age group, the number of deaths was equal between both cohorts. But the rescued infant deaths were shifted further to higher ages[4]. In the next age group, two deaths were postponed in comparison to the previous cohort.

**Figure 2** **Population model with constant birth and no migration to illustrate the extent of shifted deaths**




Source: own design

Together with the five saved deaths from age zero, the number of shifted deaths increases to seven at the beginning of the year t. The survival conditions within year t lead to a further postponement of the seven earlier saved death plus the additional rescued death at age x. This procedure can repeat for cohorts living during the analysed period. Thus, the current period conditions within in year t reduces firstly the number of deaths at age x and allows secondly the further flow of earlier rescued deaths.

We can formulate the age-specific proportion of shifted deaths at age x during the period t by the sum of all previous differences in the incidence rates between the previous and the current cohorts:

(6a) $$s(x,t) = \sum_{n=0}^{x} i(n,t-x-1) - i(n,t-x)$$

| | |
|---|---|
| $i(n,t-x)$ | Incidence rate at age n of the cohort t-x |
| $i(n,t-x-1)$ | Incidence rate at age n of the cohort t-x-1 |

[4] Theoretically, the rescued infant deaths could occurred at age 1. In this case, five deaths of age 1 were shifted into higher ages.

The solution of Eq. 5 by using Eq. 6a leads to the difference of the TMR from one (see Appendix 1). Thus, the age-specific proportion of shifted deaths within a period t includes earlier shifted death of the birth cohort t-x and the rescued deaths of the current period at age x. Another way to derive the age-specific proportion is possible by using the number of individuals who survived to the next period. The difference of the remaining survivors from the initial number of birth of the cohort t-x equals the sum of all age-specific deaths which have occurred during this cohort:

(6b) $$s(x,t) = l(0) - \frac{l_{t-x-1}(x+1)}{l(0)} - l(0) - \frac{l_{t-x}(x+1)}{l(0)}$$

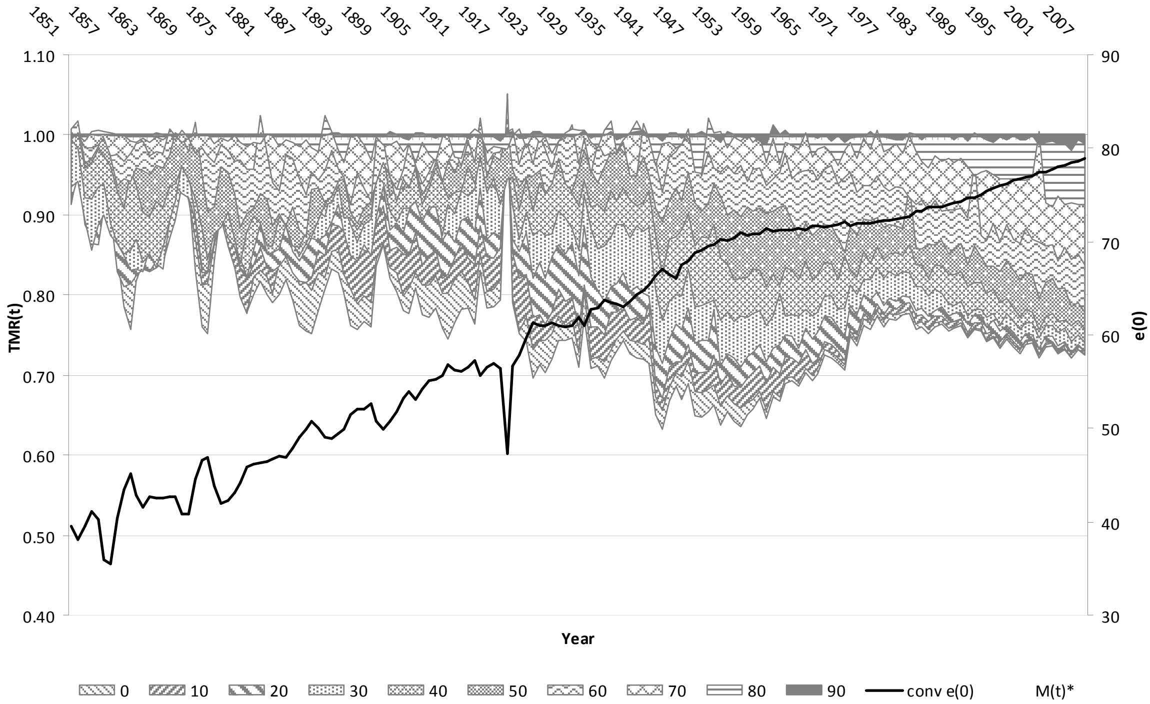
$$s(x,t) = p_{t-x}(x+1) - p_{t-x-1}(x+1)$$

$\frac{l_{t-x}(x+1)}{l(0)} = p_{t-x}(x+1)$ Proportion of survivors at age x+1 of the cohort t-x also known as survival probability from age 0 to age x+1 of the cohort t-x

The age-specific proportions of shifted deaths have several characteristics for mortality analysis. First, the proportion is a period indicator although it combines current shifted deaths as well as previous saved deaths of the cohort living during the analysed period. The reason is that only the period mortality conditions enable the further shift of earlier rescued deaths to higher ages beside the decrease or increase in observed number of deaths. However, the proportion of shifted deaths may be lower than the proportion of observed deaths. This is the case when earlier cohort mortality conditions were higher compared to the previous cohort. For illustrating this scenario, we increase the infant deaths of cohort c to 120 deaths in Figure 2 while all other age groups remain their mortality level. The number of shifted deaths at age x decreases to -17 deaths caused by the increase of the infant mortality. The negative value indicates that deaths from higher ages are pre-shifted to younger age groups compared to the previous cohort c-1. Despite the negative value of shifted deaths, their proportion remains a period indicator. It indicates only the number of shifted deaths of a cohort during the period t. Moreover, the proportion do not present the age at which the shifted deaths will be occurring in the case of previous saved deaths or have occurred in case of pre-shifted deaths.

The second characteristic of the proportion allows a detailed analysis of period mortality change. Figure 3 shows the age-decomposition of the proportion of shifted deaths among Swedish men from 1851 to 2007.

**Figure 3** **Trend of age-specific proportions of shifted deaths and the period life expectancy among Swedish men from 1851-2007**




Source: Human Mortality Database, own calculation

The last half of the 19th century is characterised by a fluctuation of all age-specific proportions of shifted deaths. At the beginning of the 20th century, the proportion of shifted death among younger ages (below age 30) started to increase what was mainly caused by the reduction of the infant mortality. The Spanish flu epidemic at 1918 reduced extremely the proportion of shifted death. Only the age groups 50-59 and the oldest age groups 80+ could significantly postpone deaths. After the shock, the survival conditions among younger ages began again to improve. While the proportion of shifted deaths among younger age groups was reduced continuously during the forthcoming periods, the middle age groups (age 30 to 59) have experienced a significant mortality reduction since 1950. This improved survival conditions were characterised by the increased proportions of postponed deaths due to the antibiotic treatment. But the proportion of shifted deaths among the middle ages decreased significantly during the 1970s. This trend was mainly driven by the increase of smoking attributable mortality and other 'man-made' diseases. At the same moment, the highest age groups (person aged 70 or older) started to experience the improvement of their survival conditions. These age groups influence mainly the recent trend of the TMR. Because of the continuous increasing of shifted deaths, the TMR has followed a decrease trend since 1985.

Figure 3 includes also the trend of the period life expectancy. The comparison shows that the trend of the period life expectancy only partly reflects the trend of the postponed deaths. There are two significant examples to illustrate the differences between both indicators. The period life expectancy at 1918 decreased rapidly as a consequence of the Spanish flu epidemic. However, the age decomposition of the shifted deaths indicates that only the ages 0-19 have experienced a worsening of their survival conditions. Their proportions of shifted deaths were negative and indicate that deaths were pre-shifted from later ages. All other age groups were also characterised by a reduction of the number of shifted deaths but the proportions were still positive. This trend of the age-specific proportion of shifted death would likely cause a stagnation of the period life expectancy than the observed rapid decline. The second example refers to the stagnated trend of the life expectancy between 1960 and 1975. Although each age group contains significant proportions of shifted deaths, the improvement in period life expectancy was very slightly. This result is very surprising because the period mortality conditions enable a continuous shift of saved deaths especially among the older age groups. Both examples show that the trend in period life expectancy does not reflect the period mortality conditions characterised by the deaths shift. According to the logic of Bongaarts and Feeney, these discrepancies are caused by tempo effects.

## 3. Age-specific proportion of shifted deaths and the tempo effect

The age-specific proportions of deaths are directly related to the discussion about tempo effects in period mortality indicators. Bongaarts and Feeney (2002, 2008) show that the conventional period life expectancy is distorted by a tempo effect even when the mortality conditions have changed. Therefore, the period life expectancy is overestimated when the survival condition has improved or is underestimated in the case of increasing mortality. Both cases show that the conventional life expectancy does not reflect the current mortality conditions. This point was controversially discussed in the previous literature. The conventional way to derive the period mortality conditions was done by the assumption that the period death rates remain constant. Under this condition, the life expectancy calculated by the observed death rates indicates the average life time of a newborn as a consequence of the current mortality conditions. On the other hand, Bongaarts and Feeney (2003, 2008) assume that the period mortality conditions can only be derived when the proportion of cohort deaths immediately reaches a value of one during the analysed period. According to their view, the period death rates have to be adjusted by the proportions of shifted deaths.

Applying the proportion of shifted deaths from Eq. 6, it is possible to derive the adjusted death rate of the 2nd kind i(x,t)* by adding the age-specific proportion of shifted death to the observed incidence rates:

(7) $$i(x,t)^* = i(x,t) + s(x,t)$$

The term 'adjusted' refers to the condition in which the total mortality rate (proportion of all cohort deaths which occur within a period) reaches exactly one. Thus, the sum of all adjusted age-specific death rates 2nd kind equals exactly one (Appendix 1). The adjusted age-specific death rate 1st kind then may be derived by the ratio proposed in Eq. 1:

(8) $$i(x,t)^* = m(x,t)^* \cdot \frac{L_{t-x}(x)}{l(0)}$$

By reformulating of Eq. 8 we get:

(9a) $$m(x,t)^* = \frac{D_{t-x}(x)^*}{L_{t-x}(x)}$$

The numerator of Eq. 9a $D_{t-x}(x)^*$ is the adjusted number of birth standardised deaths at age x of the period t. Equation 9a presents that only the number of deaths is distorted by the number of shifted deaths whilst the number of person years remains unaffected. The adjusted number of deaths may be derived directly from the difference of the cohort survivors between two ages during the period t (see Appendix 2):

(9b) $$D_{t-x}(x)^* = l_{t-x}(x) - l_{t-x-1}(x+1)$$

Both Eqs. 9a and 9b allow the reformulation of the methods for tempo-adjusted life expectancy without assuming a proportional change of mortality conditions. The first method $M_1(t)$ or CAL(t) proposed by Guillot (2008, Bongarts & Feeney 2008) sums the birth standardized person-years of each ages during a period t:

(10) $$M_1(t)^* = \sum_{x=0}^{\omega} \frac{L_{t-x}(x)}{l(0)}$$

The number of age-specific person-years is the ratio between the age-specific number of birth standardised deaths and the adjusted death rate of the 1st kind (Eq. 9a). The second method $M_2(t)$ also known as mean age at death is formulated as (Sardon 1994, Bongaarts & Feeney 2008):

$$(11) \qquad M_2(t) = \frac{1}{TMR(t)} \cdot \sum_{x=0}^{\omega} (x+0.5) \cdot i(x,t) = \sum_{x=0}^{\omega} (x+0.5) \cdot \frac{i(x,t)}{TMR(t)}$$

This method adjusts the age-specific death rates 2nd kind by the total proportion of shifted deaths. This method only leads to a tempo-adjusted life expectancy when the proportion of shifted deaths is constant over age. However, the age-specific death rate 2nd kind may be adjusted more precisely by the age-specific proportion of shifted deaths (Eq. 7):

$$(12) \qquad M_2(t)^* = \sum_{x=0}^{\omega} (x+0.5) \cdot i(x,t)^*$$

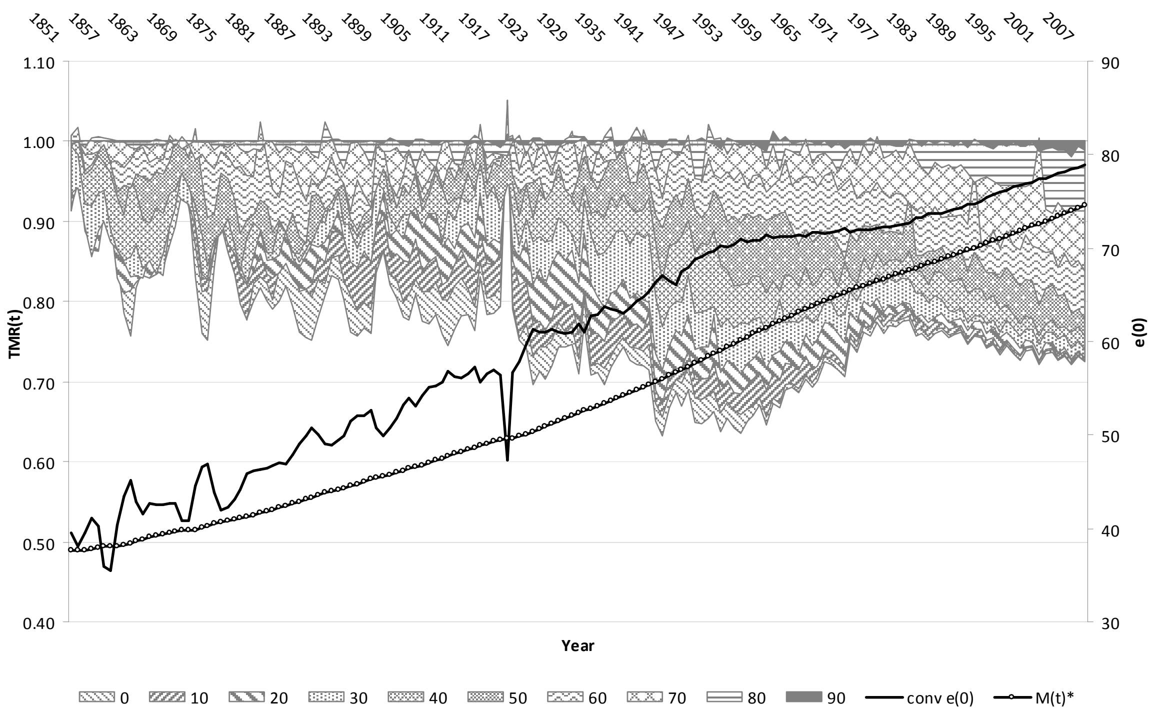
The denominator in Eq. 12 disappears because the sum of all adjusted death rates 2nd kind equals one. As can be proved in Appendix 3, $M_1(t)^*$ equals exactly $M_2(t)^*$ without any assumptions about the change in mortality condition.

The last method $M_4(t)$ adjusts the age-specific death rate by the total proportion of shifted deaths (Bongaarts & Feeney 2008). But the death rate may also be adjusted directly by the age-specific proportion of shifted deaths (Eq. 9). The age-specific incidence rates of the life table ${}^{LT}i(t)$ calculated by the adjusted death rates are similar to the adjusted incidence rates of the period. The reason is that the adjusted death rates base on a forced stationary mortality condition. Thus $M_4(t)$ may be reformulated as:

$$(13) \qquad M_4(t)^* = \sum_{x=o}^{\omega} (x+0.5) \cdot {}^{LT}i(t) \qquad \text{with} \qquad {}^{LT}i(t) = i(x,t)^*$$

The considering of the age-specific proportions of shifted deaths allows to combine all three methods for tempo-adjusted life expectancy without the proportionality assumption. Figure 4 presents the trend of the tempo-adjusted life expectancy among Swedish men beside the trend of the age decomposition of the shifted deaths and the trend of the conventional life expectancy. On the first view we can see that the tempo-adjusted life expectancy is always lower than the conventional life expectancy. This trend is caused by the amount of period shifted deaths which causes the tempo effect in each period. Another result is that the tempo-adjusted life expectancy better reflects the period mortality conditions characterised by the proportion of shifted deaths. During the Spanish flu epidemic, the tempo-adjusted life expectancy stagnated only because the most age groups had still positive proportions of postponed deaths. Furthermore, the adjusted life expectancy was characterised by a continuous increase during 1960 and 1975 whereas the conventional life expectancy stagnated. Thus, the continuous rise of the adjusted life expectancy was related to the high proportion of shifted deaths during these periods.

**Figure 4** **Trend of age-specific proportions of shifted deaths, the period life expectancy and the tempo-adjusted life expectancy among Swedish men from 1851-2007**




Source: Human Mortality Database, own calculation

## 5. Conclusion

In demographic research, the period life expectancy is an important indicator to quantify the period survival conditions. The conventional definition presumes that the life expectancy reflects the current mortality conditions. But various recent publications have critised this definition. They had shown that the period life expectancy is distorted by a temporary, disproportionate decline in the number of deaths if mortality changes in the respective period. Bongaarts and Feeney (1998, 2002) introduced the term “tempo effect” to descripe this phenomenon.

This paper presents a method to derive the age-decomposition of shifted deaths. The proportions of shifted deaths cause the tempo effect in period mortality indicators because they deflate or inflate the period mortality intensity. The method presents that the age-specific proportions of shifted deaths includes the earlier shifted deaths as well as the number of deaths which is shifted during a period. However, the proportions are still period indicators because they do not contain any information about the age or period when deaths were saved. The indicator presents only the impact of current period mortality condition on the further shift of rescued deaths. This information is helpful to reformulate the proposed methods for tempo-adjusted life expectancy. The original definition is based on a proportional change in period mortality. Indeed, the age-specific proportions of shifted deaths enable the similarity of these methods without any assumptions about the trend of survival conditions.

The empirical application presents that the age decomposition of shifted deaths is a meaningful indicator to describe the period mortality conditions. Furthermore, the comparison between the number of shifted deaths and the conventional life expectancy presents that the life expectancy only partly reflects the changes in current mortality conditions. The tempo-adjusted life expectancy, however, is on a lower level compared to the conventional one but reflects more appropriate the trend in period mortality conditions. Future analyses of period survivalship can therefore only become more authoritative through an additional tempo adjustment of conventional life expectancy.

## References


Bongaarts, J. and G. Feeney (2002). "How long do we live?" Population and Development Review **28**(1): 13-29.

Bongaarts, J. and G. Feeney (2008). Estimating mean lifetime. How long do we live? Demographic models and reflections on tempo effects. E. Barbie, J. Bongaarts and J. W. Vaupel. Berlin et al., Springer**:** 11-27.

Bongaarts, J. and G. Feeney (2008). The quantum and tempo of life-cycle events. How long do we live? Demographic models and reflections on tempo effects. E. Barbie, J. Bongaarts and J. W. Vaupel. Berlin et al., Springer**:** 29-65.

Bongaarts, J. and G. Feeney (2010). "When is a tempo effect a tempo distortion?" Genus **66**(2).

Feeney, G. (2010). "Mortality tempo:A guide for the skeptic." Comparative Population Studies - Zeitschrift für Bevölkerungswissenschaft **35**(3): 483-496.

Guillot, M. (2008). Tempo effects in mortality:An appraisal. How long do we live? Demographic models and reflections on tempo effects. E. Barbie, J. Bongaarts and J. W. Vaupel. Berlin et al., Springer**:** 129-152.

Horiuchi, S. (2008). Tempo effect on age-specific death rates. How long do we live? Demographic models and reflections on tempo effects. E. Barbie, J. Bongaarts and J. W. Vaupel. Berlin et al., Springer**:** 191-201.

Luy, M. and C. Wegner (2009). "Conventional versus tempo-adjusted life expectancy - which is the more appropriate measure for period mortality?" Genus **65**(2): 1-28.

Peto, R., A. D. Lopez, et al. (2006). "Mortality from smoking in developed countries 1950-2000, 2nd edition." from http://www.ctsu.ox.ac.uk/~tobacco/index.htm.

Sardon, J.-P. (1994). "A period measure of mortality:The example of France." Population (English Edition) **6**: 131-150.

Vaupel, J. W. (2002). "Life expectancy at current rates vs. current conditions:A reflexion stimulated by Bongaarts and Feeney's "How long do we live?"." Demographic Research **7**(Article 8): 365-377.

Wachter, K. W. (2008). Tempo and its tribulations. How long do we live? Demographic models and reflections on tempo effects. E. Barbie, J. Bongaarts and J. W. Vaupel. Berlin et al., Springer**:** 109-128.

Wilmoth, J. R. (2005). "On the relationship between period and cohort mortality." Demographic Research **13**(Article 11): 231-280.

**Appendix 1**

The total proportion of shifted deaths is the sum of all age-specific proportions of deaths (Eg. 5). The second sum indicates that age-specific proportion contains period shifted deaths and earlier shifted deaths of the cohort t-x (Eq. 6a):

(A1.1) $$S(t)=\sum_{x=0}^{\omega}\sum_{n=0}^{x} i(n,t-x-1)-i(n,t-x)$$

$$\begin{aligned} S(t) = {} & i(0,t-1)-i(0,t)+ \\ & i(1,t-2)-i(1,t-1)+i(0,t-2)-i(0,t-1)+ \\ & i(2,t-3)-i(2,t-2)+i(1,t-3)-i(1,t-2)+i(0,t-3)-i(0,t-2)+ \\ & \vdots \\ & i(w,t-w-1)-i(w,t-w)+\cdots+i(0,t-w-1)-i(0,t-w) \end{aligned}$$

$$S(t)=-i(0,t)-i(1,t)-\cdots-i(\omega,t)+i(0,t-w-1)+i(1,t-w-1)+\cdots i(\omega,t-\omega-1)$$

The Eq. A1.1 reduces to two important addends. The first addend equals the negative TMR of the period t whereas the second term leads to the CMR of the oldest cohort living during the previous period t-1:

(A1.2) $$S(t)=-TMR(t)+CMR(t-w-1)$$

The intensity of a cohort (CMR) is always one so that Eq. A1.2 can further reduce to:

(A1.3) $$S(t)=1-TMR(t)$$

Equation A1.3 presents that the sum of all age-specific proportions of shifted deaths equals the difference of the TMR from one.

## Appendix 2

The adjusted age-specific number of birth standardised deaths may be derived by the differences of the cohort survivors between two ages during a period t. Considering the proportion of shifted deaths, the proportion of survivors at age x+1 of the cohort t-x is equal the proportion of survivor at the beginning of the period minus the adjusted incidence rate at age x:

(A2.1)
$$\frac{l_{t-x}(x)}{l(0)} - i(x,t) - s(x,t) = \frac{l_{t-x-1}(x+1)}{l(0)}$$
$$p_{t-x}(x) - i(x,t) - s(x,t) = p_{t-x-1}(x+1)$$

The substitution of i(x,t) and s(x,t) from Eq. 6b we get:

(A2.2)
$$p_{t-x}(x) - p_{t-x}(x) + p_{t-x}(x+1) - p_{t-x}(x+1) + p_{t-x-1}(x+1) = p_{t-x-1}(x+1)$$
$$p_{t-x-1}(x+1) = p_{t-x-1}(x+1)$$

with $i(x,t) = p_{t-x}(x) - p_{t-x}(x+1)$

## Appendix 3

Eq. 8 presents a general relationship between the adjusted death rate of the 1st and 2nd kind by considering the age-specific proportion of shifted deaths. The use of this relationship allows to reformulating the proposed methods for tempo-adjusted life expectancy. The following calculations prove whether $M_1(t)^*$ and $M_2(t)^*$ are equal:

(A3.1)
$$\sum_{x=0}^{\omega}(x+0.5)\cdot i(x,t)^* = \sum_{x=0}^{\omega}\frac{L_{t-x}(x)}{l_{t-x}(0)}$$

The adjusted death rate 2nd kind in Eq. A3.1 may be substituted by Eq. 8 whereas the product between the adjusted death rate 1st kind and the number of person-years leads to the adjusted number of deaths:

(8)
$$i(x,t)^* = m(x,t)^*\cdot\frac{L_{t-x}(x)}{l_{t-x}(0)} = \frac{D(x,t)^*}{l_{t-x}(0)}$$

(A3.2)
$$\frac{1}{l_{t-x}(0)}\cdot\sum_{x=0}^{\omega}(x+0.5)\cdot D(x,t)^* = \frac{1}{l_{t-x}(0)}\cdot\sum_{x=0}^{\omega}L_{t-x}(x)$$

$$\sum_{x=0}^{\omega}(x+0.5)\cdot D(x,t)^{*}=\sum_{x=0}^{\omega}L_{t-x}(x)$$

Before solving the conditions of Eq. A3.2, we solve each term separately:

$$A=\sum_{x=0}^{\omega}(x+0.5)\cdot D(x,t)^{*}$$

$$B=\sum_{x=0}^{\omega}L_{t-x}(x) \qquad \text{with } A=B$$

The adjusted number of deaths Dt-x(x) in term A may be substitute by the difference of the cohort survivors between two ages during the period t (see Appendix 2):

$$\begin{aligned}A &= 0.5\cdot D(0,t)^{*}+1.5\cdot D(1,t)^{*}+\cdots(0.5+\omega)\cdot D(\omega,t)^{*}\\ &= 0.5\cdot l(0)-0.5\cdot l_{t-1}(1)+1.5\cdot l_{t-1}(1)-1.5\cdot l_{t-2}(2)+2.5\cdot l_{t-2}(2)-2.5\cdot l_{t-3}(3)+\cdots+\\ &\quad (w.5+1)\cdot l_{t-w-1}(w+1)-(w.5+1)\cdot l_{t-w}(w)+w.5\cdot l_{t-w}(w)-0\\ &= 0.5l\cdot l(0)+l_{t-1}(1)+l_{t-2}(2)+\cdots+l_{t-w-1}(w+1)+l_{t-w}(w)\end{aligned}$$

The number of person years in term B may be estimated by the differences of the survivor at age x and remaining person years of the deceased persons at age x during the period t:

$$L_{t-x}(x)=l_{t-x}(x)-0.5\cdot D_{t-x}(x)^{*}$$

The adjusted number of deaths Dt-x(x)* may be substituted again by the difference of the cohort survivors between two age during the period t (see Appendix 2):

$$\begin{aligned}B &= l(0)-0.5\cdot D(0,t)^{*}+l(1)-0.5\cdot D(1,t)^{*}+\cdots l(w)-0.5\cdot D(w,t)\\ &= l(0)-0.5\cdot l(0)+0.5\cdot l_{t-1}(1)+l_{t-1}(1)-0.5\cdot l_{t-1}(1)+0.5\cdot l_{t-2}(2)+\cdots+\\ &\quad l_{t-w-1}(w+1)-0.5\cdot l_{t-w-1}(w+1)+0.5\cdot l_{t-w}(w)+l_{t-w}(w)-0.5\cdot l(w)\\ &= l(0)-0.5\cdot l(0)+l_{t-1}(1)+l_{t-2}(2)+\cdots+l_{t-w-1}(w+1)+l_{t-w}(w)\end{aligned}$$

Both terms includes the number of cohort survivors above age 1 of the period t. Equation A3.2 reduces to:

(A3.2)
$$\begin{aligned}0.5l(0)&=l(0)-0.5l(0)\\ l(0)&=l(0)\end{aligned}$$