# AN APPLICATION OF BAYESIAN METHODS TO SMALL AREA ESTIMATES

# **OF POVERTY RATES**

Joey Campbell<sup>1</sup>

<sup>&</sup>lt;sup>1</sup> Department of Demography, University of Texas at San Antonio

## Abstract

Efforts to estimate various sociodemographic variables in small geographical areas have proven difficult with the replacement of the Census long form with the American Community Survey (ACS). ACS data products promise to begin providing up-to-date profiles of the nation's population and economy; however, the design has left researchers with significant gaps in sub-national coverage resulting in unreliable estimates for basic demographic measures. Borrowing information from neighboring areas and across time with a spatiotemporal smoothing process based on Bayesian statistical methods, it is possible to generate more stable estimates of rates for geographic areas not represented in the ACS. This research evaluates this spatiotemporal smoothing process in its ability to derive estimates of poverty rates at the county level for the contiguous United States. These estimates are compared to more traditional estimates from the Census, and error rates are calculated which substantiate the practical application of this smoothing method.

# Introduction

The decennial Census has served as the main source of detailed information on the numbers and characteristics of the U.S. population for researchers (Citro and Kalton 2007). Complete counts of people stratified by various characteristics are available at very small geographies in a variety of Census data products. The Census Bureau also utilized a very large sample to provide estimates for areas as small as block groups with the long-form sample. Information on education, employment, income, disability, commuting and other characteristics were available through the long-form every ten years. Planners used this information to develop new properties, policy makers used this information to allocate funds, and researchers used this information to investigate social processes for decades. Other household surveys provide more frequent information that details a variety of topics, but estimates are generally available at the national or statelevel or for large metropolitan areas (Citro and Kalton 2007).

The Census long-form has now been replaced by the American Community Survey (ACS). The major differences between the long-form sample and the ACS are: 1) that the ACS is conducted on a continuous basis instead of once every ten years and 2) the data are released every year. Over the last ten years, the ACS has accumulated enough responses to release statistics for all geographies available in the long-form sample. In 2010, the 5-year period summaries of the 2005-2009 responses, which had data for very small places, were released; however, there are significant gaps in subnational coverage in the ACS 1-year and 3-year period



Figure 1 ACS county-level poverty rates for the contiguous U.S., 2006.



Figure 2 ACS county-level poverty rates for the contiguous U.S., 2008.





estimates prior to the 2010 release. Areas with 65,000 or more people are sampled from 2006 forward (See Figure 1), and areas with at least 20,000 people are included in the 2008 3-year estimates (See Figure 2). The ACS contains fewer estimates for geographies prior to 2006 (See Figure 3). Researchers now have the advantage of working with yearly estimates for many sociodemographic measures, but researchers interested in sub-national demographic processes are left to work with reduced sample sizes and a loss of data.

Another issue associated with using the ACS is the errors associated with the counts. Sampling error, which accompanies every sample, is a function of sample size. In general, larger samples produce smaller sampling errors. Accordingly, ACS 1-year estimates have larger errors than 3-year estimates, which have larger errors than 5-year estimates. Many estimates from the ACS vary significantly from year to year. Often, ACS estimates conflict with other estimates from the same area. Figure 4(a) - (d)displays the estimates of the county poverty rate from both the ACS and the U.S. Census Bureau's Small Area Income and Poverty Estimates (SAIPE) for Bronx County, NY, Sevier County, TN, Multnomah County, OR, and Tulare County, CA. For the most part, estimates of poverty rates in the ACS are close to estimates from SAIPE; however, there are also some glaring discrepancies. For example, the estimates for Tulare County, CA for the ACS and SAIPE (Figure 4(d)) diverge between 2001 and 2003. In 2002, there is over a two percent difference in the estimates (10.6 for the ACS and 13 for SAIPE). For Bronx County, NY (Figure 4 (a)), ACS estimated poverty rates inexplicably jump from 28.7 percent in 2003 to 30.6 percent in 2004 only to fall again to 29.2 percent in 2005. Such variation could perhaps be possible in period estimates, but it has been argued that



Figure 6.4 Poverty Estimates from the ACS and SAIPE for selected U.S. Counties

smoothing these point estimates from the ACS could reveal a more reasonable image of the poverty situation in U.S. counties (Schmertmann 2010).

The ACS has two paramount benefits in comparison with the Census long-form sample and other available resources. First besides the Census long-form, no other data source has the amount of detailed information at the county level that is available in the ACS. Secondly, the frequency that ACS estimates are produced provides a welcome alternative to stagnate estimates when examining poverty transitions. This data source has major implications for researchers interested in subnational sociodemographic processes. For example, some multilevel studies of poverty using Census long-form data have found that there is no significant association between some structural level variables and individual risks of poverty even though they are related in theory (Cotter 2002; Partridge and Rickman 2005). The frequency of estimates from the ACS could allow for the variables at the county level to change across time. Stagnant estimates of county level rates obtained from long-form sample data products cannot change since their estimates are obtained once every ten years. If researchers are trying to measure changes, it is hard to rely on Census long-form data products to give accurate descriptions of area level characteristics the farther we move from a Census year. For instance, many studies of poverty transitions include time-varying variables to investigate how changes in these variables influence poverty transitions (Iceland 1997; McKernan, Ratcliffe and Riegg 2001; McKernan and Ratcliffe 2005). Allowing economic measures to change as household-level measures change could bring new clarity to the multi-level relationship between where an individual or family lives and how that impacts their chances of transitioning in or out of poverty.

The goal of this research is to overcome the shortcomings of the ACS estimates in order to take advantage of its benefits. To do so, estimates for the missing data for U.S. counties in the ACS must be created and verified. A substantial amount of work has been done in the field of small area estimation (Rao 2003), and within this field, Bayesian approaches to small area estimation give perhaps the most promise to providing stable and reliable estimates for missing geographies in the ACS. Using a spatiotemporal smoothing process based upon Bayesian Statistics, information from neighboring areas as well as information across time can be borrowed in order to generate reasonable estimates of rates in small areas not represented in the ACS. More specifically, six theoretical spatiotemporal models were examined in their ability to fit the ACS data, as well as their ability to provide reliable estimates of county poverty rates for the contiguous United States, and in their ability to recreate the known spatial distribution of poverty that is noted to exist in the U.S.

# Methodology

The American Community Survey (ACS) provides a count of populations with certain characteristics within U.S. counties for each year since 2000. Counts for persons below the poverty threshold, as well as total counts of persons living within each of the 3,109 counties in the contiguous United States from 2000 – 2009 were used for this analysis. Counts for 2001 – 2006 were obtained from the ACS 1-year sample, counts for 2007 and 2008 were obtained from the ACS 3-year sample, and counts for 2009 were obtained from the 5-year sample. These samples are grounded in the long-form data from Summary File 3 of the 2000 Census. These data are publicly available from the Census website (http://factfinder.census.gov/). It should be noted that this method provides

average yearly estimates for 2000 – 2006 only; thus, particular attention will be given to how the estimates of the missing geographies in these years behave. The estimates for 2007, 2008, and 2009 were purposefully chosen to determine how increasing the number of observed data can affect the estimates from the models; however, as many sociodemographic processes can change within the timeframe of the data collection for these years, smoothing these estimates may not give researchers interested in subnational sociodemographic processes the desired results as interpretation of these rates becomes cumbersome.

Poverty rates were chosen since the U.S. Census Bureau routinely estimates poverty rates at the county level through SAIPE. For more information on how SAIPE estimates county-level poverty see Bell, Basel, Cruse, Dalzell, Maples, O'Hara, and Powers (2007). County poverty estimates from SAIPE provide a tested standard to which estimates from the Bayesian models can be compared. Also, the SAIPE estimates serve as a glimpse into the changing nature of poverty and maps of estimated poverty rates can be compared to maps of SAIPE estimates to ensure that the spatial distribution of poverty is consistent across time. Finally, error rates can be calculated by systematically comparing the Bayesian estimates to the SAIPE estimates to determine which model estimates county-level poverty closest to more traditional estimation procedures.

## **Bayesian estimation**

Given that there are not many observations in the early releases of the ACS, it is virtually impossible to calculate accurate estimates of county poverty rates using traditional measures without also consulting alternative data sources. SAIPE relies on administrative data on top of ACS data to create county-level estimates of poverty. While this may work for estimating one particular county-level rate, it does not serve well to fill in the gaps in early ACS release files. Trying to incorporate outside data sources to not only estimate poverty rates using the ACS, but underemployment, educational attainment, and other factors would require data not readily available to most researchers and is beyond the scope of this study. Using a parsimonious statement about poverty in the U.S. as it relates to a specific point in both space and time could make that statement generalizable to other factors of interest to researchers looking to utilize early releases of the ACS. That is, obtaining a parsimonious model of poverty rates that accurately recreate known poverty patterns in the U.S. over the last ten years could be easily generalized to other variables besides poverty.

Bayesian statistical methods have been popularized due to their ability to address many issues related to small sample sizes and unstable estimates (McKinnon, Potter and Schmertmann 2010). Improvements in computer technology and the development of efficient sampling algorithms have made it possible to employ these methods to a variety of applied problems (Lawson 2009).

The field of Bayesian statistics is named for Thomas Bayes, an 18th Century mathematician and Protestant minister, and applications of its algorithms are based on his original theorem. In contrast to frequentist methods, which rely solely on data, Bayesian methods combine data with additional information in order to create stronger and more stable measures. Referred to as a prior  $p(\mu)$ , this additional information is combined with observed data, y, to obtain a posterior distribution  $p(\mu | y)$ . Estimates and inferences are made from this posterior distribution:

$$p(\boldsymbol{\mu} \mid \boldsymbol{y}) = \frac{p(\boldsymbol{y} \mid \boldsymbol{\mu})p(\boldsymbol{\mu})}{p(\boldsymbol{y})}$$

Priors can be informative or diffuse, and constructing a posterior distribution can be very difficult, especially if the form of the likelihood function, prior distribution, and the marginal distribution involves complex formulas or models. Integrating complex posteriors can be virtually impossible; however, it is possible to simulate a posterior distribution using Monte Carlo Markov Chain (MCMC) sampling methods (Hoff 2009).

Samples from the posterior distribution can be obtained using several different MCMC methods. This research employs Gibbs sampling, which is one of the most common methods for Bayesian estimation (McKinnon et al. 2010). When using a Gibbs sampler to simulate a posterior distribution, starting values for all parameters are first assigned. Then new samples for each parameter are made from the full conditional distribution (a conditional distribution for a parameter given everything else). That is, sample each parameter from the distribution of that parameter conditioned on everything else, making use of the most recent values and continually updating the parameter with its new value as soon as it has been sampled. Eventually the Markov chain converges so that values of all parameters are determined.

#### Spatiotemporal smoothing

To estimate county level poverty rates from the ACS, the prior distributions are derived from both previous information (Census long-form sample) and observed data from neighboring areas observed in the ACS. The long-form sample from the 2000 Census has no missing geographies, while the ACS sample from the next year (2001) has 3,109 - 18 = 3,091 missing geographies. Borrowing information across time could help elucidate the values for later years of data. Single year predictions would be especially suitable since the poverty rate does not change much from year-to-year (DeNavas-Walt, Proctor and Smith 2009). In the field of spatial statistics, a general rule of thumb is that objects closer to each other in space share similar values compared to objects farther away in space (Lichter and Johnson 2007; Voss et al. 2006). Thus, the mean and distribution of neighboring areas can be used to create a prior distribution that could strengthen area estimates that are unstable (Lawson 2009). Moreover, this information could add valuable insight to those unobserved counties in the ACS.

A hierarchical Bayesian model using the program OpenBUGS (Bayesian inference Using Gibbs Sampling) was used to incorporate prior information from neighboring areas and across time. The employed models have two levels. The first level consists of modeling the number of persons in poverty within each county using a binomial distribution. The second level comes from the prior specification of the probability that a person within a particular county is in poverty as a linear function of both space and time components.

#### Binomial Model for Poverty Rates

In examining poverty rates within each county, a count of the number of people in poverty within each county was used. Define this count as  $y_i$  and note that there are m =3,109 counties in the contiguous United States. A finite population within each small area, denoted  $n_i$  for all i, from which poverty counts are observed was also assumed. A binomial model for the count data conditional on the observed population in the areas is generally preferred for finite populations (Arató, Dryden and Taylor 2006; Lawson 2009). Therefore, given the probability that a case is below the poverty threshold  $p_i$ ,  $y_i$  is assumed to be distributed independently as

$$y_i \sim bin(p_i, n_i),$$

and that the likelihood is given by

$$L(y_i | p_i, n_i) = \prod_{i=1}^{m} {\binom{n_i}{y_i}} p_i^{n_i} (1 - p_i)^{(n_i - y_i)}.$$

A linear predictor was also chosen as a suitable link function for the probability  $p_i$ . The logit link function is the most commonly used link function (Lawson 2009) so that

$$p_i = \frac{\exp(\eta_i)}{1 + \exp(\eta_i)}.$$

Both spatial and nonspatial components are conceived in the model specification within  $\eta_i$ . This approach has been used in many diverse applications. Williams et. al. (1992) analyzed sex ratios as the number of female (or male) births compared to the total birth population count in a small area, and Morgan et. al. (2004) assumed a binomial model to investigate counts of abnormal births within small areas. Other applications include estimating low birth weights (Lawson 2009) and child mortality rates in Brazil (McKinnon et al. 2010).

## The Spatiotemporal Models

Providing a parsimonious description of the relative risk variation in space and time could be important in providing reasonable estimates for missing data. Six models that have been extensively examined in disease mapping applications were applied to small area estimation of poverty rates and were chosen because of their treatments of both space and time. They were not proposed as the best models for this purpose but offer a plausible set of models, and alternative models could be hypothesized. The goal here was to simply analyze these space-time models with respect to their ability to recreate the patterns of poverty across the contiguous United States over the last ten years. The models, outlined in more detail in Chapter 11 of Lawson (2009), represent several different specifications for the time component, while the spatially structured random effects use a conditional autoregressive (CAR) model (Lawson 2009). For each model the count of persons in poverty, y, in each county, i, at year j was modeled using a binomial distribution, and the logit of the probability,  $p_{ij}$ , was directly modeled. In general, three groups of components for  $logit(p_i)$  were considered:

$$logit(p_{ij}) = \mu_0 + A_i + B_j + C_{ij},$$

where  $\mu_0$  is an intercept (overall rate),  $A_i$  is the spatial group,  $B_j$  is the temporal group, and  $C_{ij}$  is the space-time group. Some components of the spatial group were correlated heterogeneity terms  $(u_i)$  while some were uncorrelated heterogeneity terms  $(v_i)$ . The specification of the correlated component was considered to have an intrinsic Gaussian (CAR) prior distribution (Besag, York and Mollie 1991; Lawson 2009) where the neighborhood is defined for the first neighbor only. A conventional zero-mean Gaussian prior distribution was defined by Besag et al. (1991) for the uncorrelated heterogeneity  $(v_i)$ , and was therefore assumed for these models. Diffuse inverse gamma distributions were assumed for all hyperpriors in the models; that is, the precision parameters for the normal models had diffuse inverse gamma distributions which penalize zero values but yield considerable uniformity over a wide range (Lawson 2009). An uninformative flat prior was used for the overall rate  $(\mu_0)$ . The models differ in their inclusion or exclusion of these three general components, but the priors mentioned above remain stable across the models. Other priors for parameters in any specific model are discussed in turn.

For example, Model 1 is a variant of the model from Bernardinelli, Clayton,

Pascutto, Montomoli, Ghislandi, and Songini (1995), where the probability of being in poverty was modeled as

$$logit(p_{ij}) = \mu_0 + v_i + u_i + \beta t_j$$

where  $\mu_0$  is an intercept (overall rate),  $v_i$  is an area (county) random effect,  $u_i$  is a spatially correlated heterogeneity random effect, and  $\beta t_j$  is a linear term in time  $t_j$ . The suitable priors discussed above were assumed for  $\mu_0$ ,  $v_i$ , and  $u_i$ , while an uninformative flat prior was used for  $\beta$ . With this model specification,  $A_i = v_i + u_i$ ,  $B_j = \beta t_j$ , and  $C_{ij} = 0$ .

Model 2 is exactly like Model 1, except where time was a fixed effect in Model 1 it was a random effect in Model 2. The logit specification is of the form

$$logit(p_{ij}) = \mu_0 + v_i + u_i + \tau_j,$$

where  $\tau_j$  is a separate temporal random effect and all other parameters are as they were in Model 1. An autoregressive prior distribution was used for  $\tau_j$ :  $\tau_j \sim N(\tau_{j-1}, \kappa_{\tau})$ . Subsequent models become more complex as uncorrelated heterogeneity terms for time, space-time interaction terms, and prior specifications for these parameters are changed. In this formulation,  $A_i = v_i + u_i$ ,  $B_j = \tau_j$ , and  $C_{ij} = 0$ .

Model 3 adds a temporal uncorrelated heterogeneity term to Model 3 so that the logit specification has the form

$$logit(p_{ij}) = \mu_0 + v_i + u_i + \tau_j + \xi_j,$$

where  $\mu_0$  is an intercept (overall rate),  $v_i$  is an area (county) random effect,  $u_i$  is a spatially correlated heterogeneity random effect,  $\tau_j$  is a separate temporal random effect, and  $\xi_i$  is an uncorrelated heterogeneity term for time. Much like the uncorrelated heterogeneity parameter for space ( $v_i$ ), the prior for  $\xi_j$  is assumed to be a conventional zero-mean Gaussian distribution. In this case,  $A_i = v_i + u_i$ ,  $B_j = \tau_j + \xi_j$ , and  $C_{ij} = 0$ .

Model 4 was adopted from Knorr-Held (2000) who fit a space-time model using 88 counties in Ohio of lung cancer mortality data. Their model includes a random interaction term. Here, the logit specification was defined in terms of only random effects:

$$logit(p_{ij}) = \mu_0 + v_i + u_i + \tau_j + \psi_{ij},$$

where the correlated and uncorrelated spatial components  $(u_i, v_i)$  were constant in time. A separate temporal random effect  $(\tau_j)$ , and a space-time interaction term  $(\psi_{ij})$  were also included. In this formulation,  $A_i = v_i + u_i$ ,  $B_j = \tau_j$ , and  $C_{ij} = \psi_{ij}$ . Again, an autoregressive prior was used for  $\tau_j$ , and the prior distribution for the interaction term was simply a zero mean normal.

Model 5 is the same as Model 4 except it adds the temporal uncorrelated heterogeneity term from Model 3. The logit specification has the form

$$logit(p_{ij}) = \mu_0 + v_i + u_i + \tau_j + \xi_j + \psi_{ij},$$

where  $\mu_0$ ,  $v_i$ ,  $u_i$ ,  $\tau_j$ ,  $\xi_j$ , and  $\psi_{ij}$  are as previously described and have the same suitable priors. Here,  $A_i = v_i + u_i$ ,  $B_j = \tau_i + \xi_j$ , and  $C_{ij} = \psi_{ij}$ .

Finally, Model 6 extends Model 4 by using a more complex prior for the nonseparable space-time interaction term ( $\psi_{ij}$ ). For Model 6, the logit specification has the form

$$logit(p_{ij}) = \mu_0 + v_i + u_i + \tau_j + \psi_{ij}$$

where the correlated and uncorrelated spatial components  $(u_i, v_i)$  were constant in time. A separate temporal random effect  $(\tau_i)$ , and a space-time interaction term  $(\psi_{ii})$  were also

included. In this formulation,  $A_i = v_i + u_i$ ,  $B_j = \tau_j$ , and  $C_{ij} = \psi_{ij}$ . For this model, a type II random walk interaction (Knorr-Held 2000) was defined by the prior distribution

$$p(\boldsymbol{\psi} \mid \boldsymbol{\kappa}_{\psi}) \propto \exp\left(-\frac{\boldsymbol{\kappa}_{\psi}}{2} \sum_{i=1}^{m} \sum_{j=2}^{J} (\boldsymbol{\psi}_{ij} - \boldsymbol{\psi}_{i,j-1})^{2}\right)$$

Table 1 summarizes the six models according to the general logit specification.

Table 1 Summary of Model Specification										
	Spatial Terms	<b>Temporal Terms</b>	Space-time Terms							
Model	$A_i$	$B_j$	$C_{ij}$							
1	$v_i + u_i$	$\beta t_j$	0							
2	$v_i + u_i$	$ au_j$	0							
3	$v_i + u_i$	$ au_j+igz \xi_j$	0							
4	$v_i + u_i$	$ au_j$	$\psi_{ij}$							
5	$v_i + u_i$	$ au_j+igz \zeta_j$	$\psi_{ij}$							
6	$v_i + u_i$	$ au_j$	$\psi_{ij}$							

## Results

The overall estimate of U.S. poverty in 2001 according to SAIPE was 13.74 percent. The estimates of U.S. poverty for the six models are remarkably similar. In fact, Model 1 estimates the U.S. poverty level in 2001 to be 13.97 percent, Model 2 and Model 3 estimate the U.S. poverty rate to be 13.96 percent, Model's 4 and 5 both estimate the 2001 rate to be 14.15 percent, and Model 6 estimates the U.S. poverty level in 2001 at 14.17 percent. Overall, the Bayesian models produce similar rates of those estimated by more traditional methods.

In addition to the related rates for the single year estimates, the Bayesian models reproduce the poverty trend over the last ten years. Two economic downturns translated



Figure 5 Overall U.S. Poverty Rates from SAIPE and the six Bayesian models

into a significant increase in national poverty and in many of the country's metropolitan and non-metropolitan communities (Kneebone and Garr 2010); therefore, the estimates from the different models should also produce increasing estimates of the overall poverty rate from 2000 – 2009. This trend should continue given the magnitude of the latest downturn (Edin and Kissane 2010).

Figure 5 displays the overall U.S. poverty rate as estimated by SAIPE and the six Bayesian models. While there is much variability in the estimates from SAIPE from year to year, there is a noticeable overall increasing trend from 2000 to 2009 mirroring the expected trend. Also, while the six models produce different estimates of poverty for each year, that same increasing trend is evident in each of the models. As such, the Bayesian models do well to recreate the SAIPE values for overall poverty in the U.S. in any given year (each estimate around 14 percent for 2001), and the increasing trend resulting from the two economic downturns is captured by each estimate. In addition, the estimates obtained from SAIPE are also widely variable. Using the SAIPE estimates to generate an overall poverty rate by year for the entire U.S. indicates wild shifts in the overall poverty rate over the last ten years. This is in direct conflict with official Census estimates from the Current Population Survey (CPS). The estimates obtained from this research could be a better picture of subnational poverty rates as they form a more cohesive picture of both county level and overall poverty in the U.S.

In terms of spatial distribution, the patterning of county-level poverty seen in the map of the SAIPE estimates is captured by each of the 6 models. Figures 6 through 11 display the 2001 county-level poverty estimates from Model's 1 through 6, respectively. Figure 12 displays the county-level poverty estimates from SAIPE. The patterning of poverty identified by the models also picks up on the spatial patterning of poverty identified in the literature (Lichter and Johnson 2007). That is, areas that are known to have high concentrations of impoverished residents are also identified as high poverty areas with the Bayesian estimates. Extremely poor counties, distinguished by the darkest shade in the maps, depict the same areas known to have heavy concentrations of poor residents. Namely, Appalachia, which has been identified as a persistently poor area of the U.S. (Cushing 1999; Pollard 2004), and the Native American reservations on the Great Plains are both captured as high poverty areas in the maps of the Bayesian estimates. Poverty rates are often in excess of 50 percent in communities on the Pine Ridge Indian Reservation in South Dakota (O'Hare and Johnson 2004), and are also in



Figure 6 Model 1 county-level poverty rates for the contiguous U.S., 2001.



Figure 7 Model 2 county-level poverty rates for the contiguous U.S., 2001.



Figure 8 Model 3 county-level poverty rates for the contiguous U.S., 2001.



Figure 9 Model 4 county-level poverty rates for the contiguous U.S., 2001.



Figure 10 Model 5 county-level poverty rates for the contiguous U.S., 2001.



Figure 11 Model 6 county-level poverty rates for the contiguous U.S., 2001.



Figure 12 SAIPE county-level poverty rates for the contiguous U.S., 2001.

excess of 50 percent in the Bayesian model estimates. Poverty rates are also known to be exceptionally high among African Americans in the Mississippi Delta and "Black Belt" crescent (Lee and Singelmann 2005; Parisi et al. 2005), and among Mexican-origin Hispanics in the colonias of the lower Rio Grande Valley (Saenz 1997; Saenz and Thomas 1991). Both of these areas are identified in the maps of the Bayesian estimates as having particularly high poverty rates compared to other counties in the United States. In sum, the known patterns of poverty that exist in the U.S. are adequately captured in all of the Bayesian estimates. Estimates from the Bayesian models at other time points also mimic the results produced by SAIPE. Overall, the patterning observed in maps of the SAIPE estimates for other years is reproduced in the patterning observed in maps of the Bayes estimates.

Each model seems capable of reproducing the known spatial patterning of poverty in the United States. Picking the model that produces the most accurate estimates of U.S. poverty rates is assessed by comparing the county-level estimates from each model to the estimates produced by SAIPE for each year. Mean Absolute Percent Errors (MAPE) were calculated by finding the absolute difference of each Bayesian model estimate  $(y_{ij}^{pr})$  and the SAIPE estimate  $(y_i)$  and dividing that difference by the number of counties:

$$MAPE_{j} = \frac{1}{m} \sum_{i} \left| y_{i} - y_{ij}^{pr} \right|.$$

Table 6.2 displays the error results. Overall the estimates from the models do a fair job at recreating the estimates from SAIPE. The average error rate is around 0.10. Model 2 gets closest to the SAIPE estimates in 2005 (MAPE = 0.082) while Model 1 gives the worst performance in 2009 (MAPE = 0.13).

 Table 2 Mean Absolute Percent Error Rates (MAPE) for Bayesian Estimates

Model	2001	2002	2003	2004	2005	2006	2007	2008	2009	Total
1	0.110	0.106	0.108	0.112	0.088	0.093	0.098	0.109	0.131	0.115
2	0.105	0.100	0.104	0.111	0.082	0.091	0.098	0.110	0.130	0.103
3	0.105	0.104	0.104	0.111	0.082	0.091	0.098	0.110	0.130	0.103
4	0.106	0.118	0.120	0.131	0.103	0.110	0.106	0.111	0.117	0.113
5	0.106	0.118	0.120	0.131	0.103	0.110	0.106	0.111	0.117	0.113
6	0.107	0.119	0.122	0.132	0.103	0.110	0.105	0.108	0.118	0.113

of U.S. County Poverty Rates

A noticeable pattern is evident in Table 2 in that Models 1 - 3 (the more parsimonious models) perform better than the more complex models (Models 4 - 6) in terms of recreating SAIPE estimates using single year estimates from the ACS, but this pattern reverses in later years when more data are available. To put it another way, the more data that are available, the better more complex models perform. The error rates for Models 1 - 3 are lower than the error rates for Models 4 - 6 for 2001 - 2006 estimates of county poverty rates, but Models 4 - 6 have lower error rates than Models 1, 2, and 3 in 2009 where full data are available from the ACS 5-year estimates. Model 6, the most complex model, is best at recreating SAIPE estimates for 2008 (MAPE = 0.108).

Models 2 and 3 produce the best estimates in terms of reproducing SAIPE estimates, and the estimates produced by both models are very similar. Often rates differ only by thousandths of a percent. Model 2 is chosen as the better(Abramsky 2009) model since its specification was more parsimonious than Model 3 while producing similar estimates. In fact, the highest posterior density interval for the added uncorrelated



Figure 13 Poverty Estimates for Barton County, KS.

heterogeneity parameter for time in Model 3 includes zero indicating that it does not add any additional information compared to Model 2.

Figure 13 displays the poverty rate for Barton County, KS as estimated by Model 2, SAIPE, and the ACS. Barton County only appeared in the ACS after 2007, and even through the estimates of its poverty rate were averaged over 3 years for 2007 and 2008 and 5 years for 2009 there is still a wide shift observed. According to the ACS, Barton County had an average poverty rate of 11.84 from 2005 – 2007, but that estimate increases to an average of 14.13 percent from 2006 – 2008. These varying estimates from overlapping time-periods give some indication to the wide variability in estimates from the ACS. Also, the poverty estimates from SAIPE for Barton County are exhibit wide shifts which is counter to known patterns of poverty. One of the other advantages to this

Bayesian estimation technique is that these wide shifts are generally smoothed after information from neighboring counties and information from previous time points are also considered. Model 2 gives a smoothed estimate of Barton County poverty over the last ten years and is less variable than the SAIPE estimate.

### Discussion

Researchers interested in subnational demographic processes should be encouraged by the ability of these Bayesian models to reproduce patterns of poverty known to exist in the United States. Although very little data exists in the ACS from early release files it is possible to create reasonable estimates of rates in these years without consulting additional data sources. Researchers working with Census long-form data must now rely on ACS data products to help answer their research questions, and this analysis demonstrates that accurate and reasonable estimates of missing values can be obtained with relative ease, which could prove fruitful for researchers waiting until the ACS obtains enough samples to produce reliable estimates for all geographies in the U.S. Consequently, the construction of reliable estimates of poverty for small geographical areas is easily expandable to other measures. The purpose of this research was to present a method which would allow for more reliable and accurate measures of poverty rates for all counties in the contiguous United States. Using a Bayesian approach that borrows information from neighboring counties as well as county estimates across time, estimates of county poverty rates have become demonstrably more stable and dependable. These estimates and this method can be employed by those investigating sociodemographic processes at the local level, eventually resulting in a smoother transition for researchers from analyzing Census long-form data to new ACS data products.

# References

- Abramsky, S. 2009. "Solving America's hunger crisis: The US government must make sweeping social reforms to help the millions of Americans forced to go hungry." in *The Guardian*. London.
- Arató, N.M., I.L. Dryden, and C.C. Taylor. 2006. "Hierarchical Bayesian modelling of spatial agedependent mortality." *Computational Statistics and Data Analysis* 51(2):1347-1363.
- Bell, W., W. Basel, C. Cruse, L. Dalzell, J. Maples, B. O'Hara, and D. Powers. 2007.
  "Use of ACS Data to Produce SAIPE Model-Based Estimates of Poverty for Counties." edited by U.S.C. Bureau. Washington DC: U.S. Census Bureau.
- Bernardinelli, L., D. Clayton, C. Pascutto, C. Montomoli, M. Ghislandi, and M. Songini. 1995. "Bayesian-Analysis Of Space-Time Variation In Disease Risk." *Statistics in Medicine* 14(21-22):2433-2443.
- Besag, J., J.C. York, and A. Mollie. 1991. "Bayesian Image Restoration, with two applications in spatial statistics (with discussion)." *Annals of the Institute of Statistical Mathematics* 43:1-59.
- Citro, C.E.and G. Kalton. 2007. "Using the American Community Survey: Benefits and Challenges." Washington, DC: The National Academy Press.
- Cotter, D.A. 2002. "Poor people in poor places: Local opportunity structures and household poverty." *Rural Sociology* 67(4):534-555.
- Cushing, B. 1999. "Migration and persistent poverty in rural America." in *Migration and Restructuring in the U.S.*, edited by K. Pandit and S.D. Withers. Lanham, MD Rowmen and Littlefield Press.
- DeNavas-Walt, C., B.D. Proctor, and J.C. Smith. 2009. "Income, Poverty, and Health Insurance Coverage in the United States: 2008." Washington DC: U.S. Census Bureau.
- Edin, K.and R.J. Kissane. 2010. "Poverty and the American Family: A Decade in Review." *Journal of Marriage and the Family* 72(3):460-479.
- Hoff, P.D. 2009. "A First Course in Bayesian Statistical Methods." New York, NY Springer.
- Iceland, J. 1997. "The dynamics of poverty spells and issues of left-censoring." Ann Arbor, MI: Population Studies Center, University of Michigan.

- Kneebone, E. and E. Garr. 2010. "The Suburbanization of Poverty: Trends in Metropolitan America, 2000 to 2008." in *METROPOLITAN OPPORTUNITY SERIES*: Brookings Institution.
- Knorr-Held, L. 2000. "Bayesian modelling of inseparable space-time variation in disease risk." *Statistics in Medicine* 19(17-18):2555-2567.
- Lawson, A.B. 2009. *Bayesian Disease Mapping*. Boca Raton, FL: Chapman & Hall/ CRC.
- Lee, M.A.and J. Singelmann. 2005. "Welfare Reform Amidst Chronic Poverty in the Mississipi Delta." in *Population Change and Rural Society*, edited by W.A. Kandel and D.L. Brown. Dordrecht, Netherlands: Springer.
- Lichter, D.T.and K.M. Johnson. 2007. "The changing spatial concentration of America's rural poor population." 72(3):331-358.
- McKernan, S.-M., C.R. Ratcliffe, and S. Riegg. 2001. "Transition Events in the Dynamics of Poverty: A Review of Issues and Results." Washington, D.C.: The Urban Institute.
- McKernan, S.M.and C. Ratcliffe. 2005. "Events that trigger poverty entries and exits." *Social Science Quarterly* 86(5):1146-1169.
- McKinnon, S., J.E. Potter, and C.S. Schmertmann. 2010. "Municipality-level estimates of child mortality for Brazil: A new approach using Bayesian Statistics. ." Population Research Center, University of Texas at Austin.
- Morgan, O.W.C., M. Vrijheid, and H. Dolk. 2004. "Risk of Low Birth Weight near EUROHAZCON Hazardous Waste Landfill Sites in England." Archives of Environmental Health 59(3):149 - 151
- O'Hare, W.P.and K.M. Johnson. 2004. "Child Poverty in Rural America." *Repots on America* 4(March):1-19.
- Parisi, D., S.M. Grice, M. Taquino, and D.A. Gill. 2005. "Community Concentration of Poverty and Its Consequences on Nonmetro County Persistence of Poverty in Mississipi." *Sociological Spectrum* 25(4):469-483.
- Partridge, M.D.and D.S. Rickman. 2005. "Persistent Pockets of Extreme American Poverty: People or Placed Based?" in *RPRC 05-01, RUPRI Rural Poverty Research Center.*
- Pollard, K.M. 2004. "A 'New Diversity': Race and Ethnicity in the Appalachian Region." in *Demographic and Socioeconomic Change in Appalachia*. Washington, DC: Population Reference Bureau.

Rao, J.K. 2003. Small Area Estimation. Hoboken, NJ: John Wiley & Sons Inc.

- Saenz, R. 1997. "Ethnic Concentration and Chicano Poverty: A Comparative Approach." Social Science Research 26(2):205-228.
- Saenz, R.and J.K. Thomas. 1991. "Minority Poverty in Nonmetropolitan Texas." *Rural* Sociology 56(2):204-223.
- Schmertmann, C. 2010. "Baysian Models for Temporal Smoothing of American Community Survey Estimates." in *Southern Demographic Association Annual Conference*. Knoxville, TN.
- Voss, P., D.D. Long, R.B. Hammer, and S. Friedman. 2006. "County Child Poverty Rates in the U.S.: A Spatial Regression Approach." *Population Research and Policy Review* 25(4):369-391.
- Williams, F., A.B. Lawson, and O. Lloyd. 1992. "Low sex ratios of births in areas at risk from air pollution from incinerators, as shown by geographical analysis and 3dimensional mapping." *International Journal of Epidemiology* 21(2):311-319.