

**Using Cohort Change Ratios to Estimate Life Expectancy  
in Populations with Negligible Migration: A New Approach\***

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\* An earlier version of this paper was presented at the 45<sup>th</sup> Actuarial Research Conference, which was held at Simon Fraser University, Burnaby, British Columbia, Canada, July 26-28, 2010. The authors are grateful for comments made by participants, including Professor Yvonne Chueh.

# **Using Cohort Change Ratios to Estimate Life Expectancy in Populations with Negligible Migration: A New Approach**

## **Abstract**

Census survival methods are the oldest and most widely applicable methods of estimating adult mortality and for populations with negligible migration they can provide excellent results. The reason for this ubiquity is threefold: (1) their data requirements are minimal in that only two successive age distributions are needed; (2) the two successive age distributions are usually easily obtained from census counts; and (3) the method is straightforward in that it requires neither a great deal of judgment nor “data-fitting” techniques to implement. This ubiquity is in contrast to other methods, which require more data, as well as judgment and, often, data fitting. In this short note, the new approach we demonstrate is that life expectancy at birth can be computed by using census survival rates in combination with an identity whereby the radix of a life table is equal to 1 ( $l_0 = 1.00$ ). We point out that our suggested method is less involved than the existing approach. We compare estimates using our approach against other estimates and find it works reasonably well. As well as some nuances and cautions, we discuss benefits in using this approach to estimating life expectancy, including the ability to develop estimates of average remaining life at any age. We believe that the technique is worthy of consideration for use in estimating life expectancy in populations that experience negligible migration.

**Keywords:** incomplete data, estimation, census data

As noted in *Methods for Estimating Adult Mortality from Census Data* (United Nations (2002: 5), “Census survival methods are the oldest and most widely applicable methods of estimating adult mortality...(and can) provide excellent results (for) populations that experience negligible migration...” The reason for the ubiquity of this approach is threefold: (1) data requirements are minimal in that only two successive age distributions are needed; (2) the two successive age distributions are usually easily obtained from census counts; and (3) the method is straightforward in that it requires neither a great deal of judgment nor “data-fitting” techniques to implement. This ubiquity is in contrast to other methods, such as “Model Life Tables”, which require more data, as well as judgment and, often, data fitting (United Nations, 1982: 16-27). Our purpose in this paper, however, is not to debate the relative merits of these and other approaches (e.g. Swanson,1989; Swanson, McGehee, and Hoque, 2009; Swanson and Palmore, 1976; Swanson, Palmore and Sundaram, 1977). Our purpose here is to simply demonstrate a new way to calculate life expectancy from census survival rates, one that is less involved than the existing method.

Census survival methods require two age distributions for a population at two points in time, which generally are two successive census enumerations. Ideally, the interval between the census enumerations (e.g., 10 years) is either equal to the width of the age groups (e.g., the age groups are given in ten year increments, 0-9, 10-19,...75-84, 85+) or a whole number multiple thereof (e.g., the age groups are given in five year increments, 0-4, 5-9,...,80-84, 85+ ), through the final open-ended age group (e.g., 85+).

The United Nations (2002: 6) shows that using the census survival method, that the expectation of life at age  $x$  can be computed as

$$e_x = (T_x/l_{(n/2)}) / (l_x/l_{(n/2)}) = T_x / l_x \quad [1]$$

where

$x$  = age

$n$  = the width of the age groups (up to, but not including the terminal, open-ended age group)

$e_x$  = life expectancy (average years remaining) at age  $x$

$T_x$  = Total person years remaining to persons age  $x$

$l_x$  = number reaching age  $x$

$l_{(n/2)}$  = persons aged  $x$  to  $x+n$  are assumed to be concentrated at the mid-point of the age group

and 
$$l_{(x+2n/2)}/l_{(x-n/2)} = P2_{(x,n)}/P1_{(x-n,n)} \quad [2]$$

where

$P2_{(x,n)}$  = the number of persons counted in the second census in age group  $x$  to  $x+n$

$P1_{(x-n,n)}$  = the number of persons counted in the first census In age group  $x-n$  to  $x$

In general, then, the life-table probability of surviving from the mid-point of one age group to the next ( $l_{(x+2n/2)}/l_{(x-n/2)}$ ) is approximated by the census survival ratio

$(P2_{(x,n)}/P1_{(x-n,n)})$ . Continuing, the same United Nations Manual (2002: 5-6) shows that the cumulative multiplication of the probabilities shown in [2] gives the conditional survival schedule  $l_x/l_{(n/2)}$ . From the conditional  $l_x$  values given by [2] the conditional estimates of the number of person years lived in each age group ( ${}_nL_x$ ) can be calculated as

$${}_nL_x/l_{(n/2)} = (n/2) * [(l_x/l_{(n/2)} + l_{(x+n)}/l_{(n/2)})] \quad [3]$$

where

${}_nL_x$  = number of person years lived in each age group

Given a value of  $T_x/l_{(n/2)}$  for some initial old age  $x$ , the UN shows that total remaining years expected at age  $x$  ( $T_x$ ) values can be calculated as:

$$T_{(x-n)}/l_{(n/2)} = T_x/l_{(n/2)} + {}_nL_{(x-n)}/l_{(n/2)} \quad [4]$$

This leads us back to equation [1], so that the expectation of life at age  $x$  using the United Nations (2002) approach is:

$$e_x = (T_x/l_{(n/2)}) / (l_x/l_{(n/2)}) = T_x / l_x$$

In our approach, we start by noting that when the radix of a life table is equal to 1 ( $l_0 = 1.00$ ) life expectancy at birth can be computed directly from the expression:

$$e_0 = S_0 + (S_0 * S_1) + (S_0 * S_1 * S_2) + \dots + (S_0 * S_1 * S_2 * \dots * S_x) \quad [5]$$

where

$e_0$  = life expectancy at birth

$S_0$  = survivorship from  $t=0$  (e.g., birth) to  $t=1$  (e.g., age 1)

$S_1$  = survivorship from  $t=1$  (e.g., age 1) to  $t=2$ (e.g., age 2)

and so on through  $S_x$

and  $S_x = {}_1L_x / {}_1L_{(x-n)}$

Equation [5] is set up for single year age groups. However, we can generalize it to other age groups:  ${}_nS_x = {}_nL_x / {}_nL_{(x-n)}$ , so that

$$e_0 = {}_nS_0 + ({}_nS_0 * {}_nS_1) + ({}_nS_0 * {}_nS_1 * {}_nS_2) + \dots + ({}_nS_0 * {}_nS_1 * {}_nS_2 * \dots * {}_nS_x) \quad [5.a]$$

As equation [5] and equation [5.a] both imply, the fundamental life table function is inherent in our method. That is via the  ${}_nS_x$  values, we have  ${}_nq_x$  values. The Appendix shows a derivation of the relationship between survivorship rates and life expectancy as shown in equation [5] and generalized to equation [5.a].

We also apply census survival rates, although we prefer to use the more general term “cohort change ratios” (CCRs). CCRs enter into a range of measures and applications, such as the "Hamilton-Perry Method," which is often used for doing short term population forecasts of small areas such census tracts (Smith, Tayman, and Swanson, 2001:153-158). Following Smith, Tayman, and Swanson (2001: 155) and using notation from equation [2], a CCR can be generally defined as:

$${}_nCCR_x = P2_{(x,n)} / P1_{(x-n,n)} \quad [6]$$

Our new approach is the result of combining, on the one hand, either equation [5] or [5.a] for computing life expectancy with, on the other hand, equation [6] in order to estimate  $e_x$ . Starting with

$${}_nS_x = {}_nL_x / {}_nL_{(x-n)} \approx P2_{(x,n)} / P1_{(x-n,n)} \quad [7]$$

We have, as shown in equation [5.a]

$$e_0 = {}_nS_0 + ({}_nS_0 * {}_nS_1) + ({}_nS_0 * {}_nS_1 * {}_nS_2) + \dots + ({}_nS_0 * {}_nS_1 * {}_nS_2 * \dots * {}_nS_x)$$

As is the case with the more involved United Nations (2002) approach, this approach will only work for populations for which migration is negligible, but there are many areas of interest around the world where this is the case, or approximately so (United Nations, 2002). The world as a whole meets this requirement. Countries with negligible migration include North Korea and Burma, among others. Other such populations are found in the historical record - the former Soviet Union, Albania from 1950 to 1980, and the Peoples Republic of China from 1950 through 1970, for example. Still others may be defined by race and ethnicity or other 'rules' of membership (e.g., Indigenous Populations in Australia and Canada, Native Hawaiians; native-born populations).

Broadly speaking, the method can be applied to any population subject to renewal through a single increment (birth) and extinction through a single decrement (death), where there are at least two successive census counts that provide the population by age. We also note that unlike the UN method, the approach we take can be used to yield estimates of life expectancy at birth. Moreover, like the UN approach, this one is not subject to the limitations imposed by stationary or even stable population requirements.

We have developed life expectancy estimates directly from cohort change ratios constructed for the world as a whole and Burma. We compare our life expectancy estimates to estimates for the world as a whole and Burma for the period 1950-55 to 2045-50 (Census Bureau, 2010). The comparison of our estimates of  $e_0$  for the world as a whole and those of the United Nations are found in Table 1. The Mean Absolute Percent Difference between our estimates and those made by the United Nations over the entire period is 3.28% while the Mean Algebraic Percent Difference is 0.68%. Both of the summary measures indicate rather close agreement between the two sets of estimates and the latter suggests that our estimates are only slightly upwardly biased relative to those of the United Nations.

(TABLE 1 ABOUT HERE)

Table 2 shows  $e_0$  estimates for Burma for the period 1975-80 to 2005-2010. As the table shows all of our  $e_0$  estimates are less than those done by the U.S. Census Bureau, which means the mean Algebraic Percent Difference of -4.50% is the same as the Mean Absolute Percent Difference. Our estimates of life expectancy at birth remain almost constant at age 60 from 1995-2000 to 2005-10 while those available from the Census Bureau increase from 61 to 65 years. However, we again find that the summary measures suggest reasonably close agreement.

(TABLE 2 ABOUT HERE)

As well as some nuances (e.g., converting CCRs into survival ratios may require additional refinements) and cautions (e.g., the population data by age may be faulty), we find benefits in using this approach to estimating life expectancy, including the ability to develop estimates of average remaining life at any age (not shown here). We suggest that



the technique is worthy of consideration for use in estimating life expectancy in for populations experiencing negligible migration, given the cautions we discuss. As such, we believe that this new approach adds another dimension to census survival methods, which as we noted at the outset are "...the oldest and most widely applicable methods of estimating adult mortality...(and can) provide excellent results (for) populations that experience negligible migration..." (United Nations (2002: 5).

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## Appendix. The Relation between Survival Rates and Life Expectancy

Any particular set of age-specific survival rates implies a specific life expectancy. As an example using a complete (single year life table where  $x =$  a given age), the relationship between a set of survival rates,  $S_x$  and  $L_x$ , the corresponding entries in the "years lived" column of the life table is  $S_x = L_x/L_{x-1}$  for ages 1 and over, while  $S_0 = L_0$  for survivors from birth to the age zero age group. Thus, in a life table with a radix = 1.0, life expectancy at birth can be expressed as  $e_0 = L_1 + L_2 + L_3 + \dots, L_x$ . That is, life expectancy can be expressed as the sum of the  $L_x$  values.

It is readily seen that:

$$L_0 = S_0$$

$$L_1 = S_0 * S_1$$

$$L_2 = S_0 * S_1 * S_2$$

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$$. L_x = S_0 * S_1 * S_2, \dots, S_{z-1} * S_x$$

Substituting  $S_x$  for  $L_x$  in the preceding yields Equation [5]:

$$e_0 = S_0 + (S_0 * S_1) + (S_0 * S_1 * S_2) + \dots + (S_0 * S_1 * S_2, \dots, * S_x)$$

And Equation [5] can be generalized to apply to an abridged life table and expressed as Equation[5.a]

$$e_0 = {}_nS_0 + ({}_nS_0 * {}_nS_1) + ({}_nS_0 * {}_nS_1 * {}_nS_2) + \dots + ({}_nS_0 * {}_nS_1 * {}_nS_2, \dots, * {}_nS_x)$$

**Table 1. Comparison of Life Expectancy Estimates for the World as Whole calculated from Cohort Change Ratios (during each period), 1950-55 to 2045-50 shown in Equation [7] and Equation[5.a] with estimated values available from the US Census Bureau (2010).**

YEARS	e <sub>0</sub> Calculated from UN Census S <sub>x</sub> values using [6] and [4]*	e <sub>0</sub> Calculated from UN Census S <sub>x</sub> values using [6] and [4]*	e <sub>0</sub> Calculated from UN Census S <sub>x</sub> values using [6] and [4]*	UN Life Expectancy(e <sub>0</sub> )		
	total e <sub>0</sub>	male e <sub>0</sub>	female e <sub>0</sub>	total e <sub>0</sub>	male e <sub>0</sub>	female e <sub>0</sub>
1950-55	51.80	50.28	53.38	46.6	45.2	48
1955-60	53.95	52.01	56.01	49.5	48.1	50.9
1960-65	56.00	54.61	57.35	52.4	51	53.7
1965-70	58.43	56.69	60.15	56.1	54.6	57.6
1970-75	60.40	58.89	61.85	58.2	56.6	59.8
1975-80	61.77	59.73	63.81	60.2	58.3	62
1980-85	62.61	60.48	64.77	61.7	59.7	63.7
1985-90	63.49	61.27	65.76	63.2	61.2	65.2
1990-95	64.05	61.55	66.65	64	61.9	66.2
1995-2000	64.90	62.39	67.54	65.2	63	67.4
2000-05	65.80	63.35	68.34	66.4	64.2	68.6
2005-10	66.65	64.30	69.07	67.6	65.4	69.8
2010-15	67.65	65.42	69.93	68.9	66.7	71.1
2015-20	68.57	66.36	70.83	70.1	67.9	72.3
2020-25	69.38	67.17	71.65	71.1	68.9	73.4
2025-30	70.14	67.91	72.42	72.1	69.9	74.4
2030-35	70.81	68.58	73.09	73.1	70.8	75.4
2035-40	71.43	69.19	73.72	73.9	71.7	76.3
2040-45	72.02	69.79	74.31	74.8	72.5	77.1
2045-50	72.59	70.37	74.88	75.5	73.3	77.9

The calculations are found in a worksheet available from the authors.

\* e<sub>0</sub> is estimated by summing the products of adjacent S<sub>x</sub> values as follows,

$$e_0 = S_1 + S_1 * S_2 + S_1 * S_2 * S_3 + S_1 * S_2 * S_3 + \dots + S_1 * S_2 * S_3 * \dots * S_k$$

**Table 2. Comparison of Life Expectancy Estimates For Burma Calculated from Cohort Change Ratios (during each period), 1975-80 to 2005-10 as shown in Equation [7] and Equation[5.a] with estimated values available from the US Census Bureau (2010).**

Source of Life Expectancy Estimate/Year	1975-80	1980-85	1985-90	1990-95	1995-2000	2000-2005	2005-2010
Life expectancy at birth (US Census, 2010)	54	56	56	59	61	63	65
Estimated Life Expectancy from CCRs	49.99	52.47	55.96	56.97	60.09	60.82	60.99