Joint Probabilistic Projection of Female and Male Life Expectancy

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Abstract

We propose a methodology for obtaining joint probabilistic projections of female and male life expectancy at birth (e_0) for all the countries in the world, building on previous work by Chunn et al. (2010) which produces probabilistic projections of e_0 for each sex independently. Our approach is to project the *gap* between female and male e_0 based on projections of female e_0 from the Chunn et al. (2010) model. Indeed, our autoregressive model for the gap in a future period for a particular country is a function of female life expectancy and a t-distributed random perturbation. The model we propose is intended to take into account mortality data limitations, be comparable across countries, and account for possible shocks such as armed conflicts. We estimate all parameters based on life expectancy estimates for the period 1950-2010 reported in the United Nations' World Population Prospects 2008. We evaluate our model using out-of-sample projections for the period 1995-2010, and compare our results to competing approaches.

1 INTRODUCTION

Every two years, the Population Division of the United Nations ("UN") publishes the World Population Prospects ("WPP"), reporting population estimates and projections for all the countries in the world. Experts then use these projections either directly to monitor population goals, or as inputs into various economic or environmental models. As such, the UN's population projections form the basis of many nations' and regions' development plans (Heilig et al. 2010).

Since 1980, every version of the WPP produced three projection variants: low, medium and high. The medium variant is intended to represent the most likely population size given past trends and expert knowledge. The low and high variants are then generated using different fertility assumptions: total fertility is taken to be half a child either below or above that used to generate the medium scenario. Thus, the variation in fertility allowed by this method is insensitive to either the level of fertility in a particular country or its past trends (Heilig et al. 2010).

Given the inability of deterministic population projections both to fully take into account the variability in demographic processes across countries and to indicate a *range* of future population outcomes, the the UN has sought to transition to probabilistic population projection methods. Since WPP projections are based on the cohort-component method, probabilistic projections of the main demographic processes affecting national populations (fertility, mortality and migration) are required to produce probabilistic population projections for all the world's countries.

Probabilistic projections of fertility for all the countries in the world have already been developed by Alkema et al. (2011) and implemented in the latest WPP 2010. Indeed, the medium variant in WPP 2010 corresponds to the median of the many possible future fertility paths for each country (100,000) generated by the Alkema et al. (2011) model.

In this work, we are concerned with the projection of mortality. As mentioned in a recent comprehensive review of population projection methods, the simplest method of forecasting mortality is to extrapolate life expectancy and use model life tables to obtain the age pattern required for use in cohort-component methods (Booth 2006). Given lacunae and frequent inaccuracies in data for many countries, this simplest method is the one employed by the United Nations.

A probabilistic approach to projecting life expectancy for males and females independently has been developed by Chunn et al. (2010) and is an extension of the UN's current method. It models expected gains in life expectancy over a period as a random walk with a nonconstant drift, where the drift term is a nonlinear function of current life expectancy and reflects varying rates of improvement for countries at different levels of life expectancy. Using a hierarchical model structure (Bayesian Hierarchical Model or BHM) allows for the estimation of the rate of improvement in life expectancy for a country using past data from that country while also taking into account observed past patterns in all other countries.

While BHM has the advantage of using information for all countries to inform projections of life expectancy of males or females for a particular country, it is insensitive to the *relationship* between male and female life expectancy in that country. In other words, this model - like a multitude of multitude of other mortality forecasting models in existence (Soneji & King 2011, Shang et al. 2011, Cairns et al. 2011, Bongaarts 2009, Haberman & Renshaw 2008, Hyndman & Shahid Ullah 2007) - is not designed to produce joint projections of male and female life expectancy at birth, which are required to obtain fully probabilistic projections of life expectancy for both sexes and avoid crossovers that are not expected to occur in the future.

The work described herein is concerned with the joint probabilistic projection of male and female life expectancy. It proposes to unify projections of life expectancy for the two sexes by projecting the ***gap*** in life expectancy between them.[1] Thus, projections of female life expectancy are generated by BHM, and then combined with projections of the gap to produce projections of male life expectancy.

In focusing on the gap in life expectancy between females and males, this paper adds to a large body of literature studying comparative mortality patterns of the two sexes. Several explanations have emerged for the difference in patterns, from the biological, to the behavioral and socioeconomic (Oksuzyan et al. 2008; Rogers et al. 2010; Trovato & Lalu 1998; Pampel 2005; Elo & Drevenstedt 2005; Trovato & Heyen 2006; Trovato & Lalu 2007). In addition, many have commented on the recent narrowing of the gap in high-income countries (Trovato & Lalu 1996, 1998; Bobak 2003; Conti et al. 2003; Meslé 2004; Elo & Drevenstedt 2005; Gómez-Redondo & Boe 2005; Pampel 2005; Preston & Wang 2006; Trovato & Heyen 2006; Glei & Horiuchi 2007; Trovato & Lalu 2007; Emslie & Hunt 2008; Kołodziej et al. 2008; Oksuzyan et al. 2008; Staetsky 2009; Staetsky & Hinde 2009; Trovato & Odynak 2011).

While several authors have provided opinions about the probable trajectory of the gap in high-income countries (Meslé 2004; Trovato & Lalu 2007; Rogers et al. 2010), very few have attempted to make explicit projections (Pampel 2005; Preston & Wang 2006). When they did so, they focused solely on data-rich high-income countries. We are aware of only two studies that study the gap in life expectancy in developing countries: they show that conflicts (Plümper & Neumayer 2006) and natural disasters (Neumayer & Plümper 2007) have different effects on the life expectancy of the two sexes, and thus affect the gap between them.

We propose to model the gap in life expectancy between females and males for all the countries in the world that are not in the midst of a generalized HIV/AIDS epidemic. After briefly discussing the data we use, we describe the model in Section 3. We then assess the performance of our model and possible competing models via cross-validation using data from

[1] In our framework, a positive gap indicates excess life expectancy for females over males.

1950-1995 to project life expectancy over the period 1995-2010. In Section 5, we discuss our projections for four countries chosen for their ability to showcase the possible future trends in the gap between female and male life expectancy: continued decline in the gap for a country in which the decline has already been observed (Ireland), decline in the gap for a country in which the decline has not yet been observed (Guatemala); continued rise, followed by a decline (Laos), and continued rise over the projection period (Cambodia). Finally, we review our approach and results and propose extensions in Section 6.

2 DATA

For the purposes of our work, we use the WPP 2008 estimates of past life expectancy measures, as well as the WPP 2008 projections of life expectancy for the period 2010-2050 for all the countries in the world.

It is widely recognized that there are significant lacunae and inaccuracies in mortality data for many countries. Indeed, according to the 2005 UN World Mortality Report, since 1990, only 56% of 196 countries with 100,000 inhabitants or more in 2009 have "reliable" or "fairly reliable" vital statistics for adult mortality, and 7% of countries completely lack recent data for the estimation of adult mortality. Countries for which data is unreliable are concentrated mostly in Asia (44% of all Asian countries) and Africa (91% of all African countries). Unreliable data results in frequent updates of estimates of past life expectancy: between WPP 2008 and WPP 2010 estimates, life expectancy estimates 1950-2005 for 79 countries have changed by more than a year in total. Modifications range from -8.22 to 21.14 years in a quinquennium for men and -8.07 to 8.14 years in a quinquennium for women, resulting in differences in gap measurements of up to 21.32 years (Iran 1980-1985). Naturally, as estimates of past data improve, so should the projections based on that data (Keilman 2001).

It is well known that mortality patterns are significantly affected by the HIV/AIDS epidemic, which has been a major source of mortality in the last twenty years. We therefore exclude from our study countries with a generalized HIV/AIDS epidemic. A country is considered to have a generalized epidemic when: (a) HIV is established in the general population; (b) the epidemic could be sustained via sexual networking in the general population independent of sub-populations at higher risk for infection; and (c) HIV prevalence is consistently over 1%.[2] We thus concentrate on 158 countries with 100,000 inhabitants or more in 2009 and no generalized HIV/AIDS epidemic.

[2]Classification of countries with a generalized HIV/AIDS epidemic was based on HIV/AIDS fact sheets published jointly by WHO, UNAID and UNICEF (2008).

The data for male and female life expectancy at birth in these 158 countries for the period 1950-2010 is shown in Figure 1 below. We note several patterns: both male and female life expectancy at birth have been increasing over time, with median female life expectancy in the last period (76.52 years) being higher than male (70.86 years). The variability in the data on life expectancy for these 158 countries has also decreased over time. The trend in the median gap between female and male life expectancy has been increasing from 1950-1955 to approximatley 1990-1995, with a slight decline thereafter. In 2005-2010, the median gap in life expectancy is estimated at 4.86 years, with an interquartile range of (3.88, 6.61). We also note that there has been some reduction in variability in the gap in life expectancy in the countries of the world; however, in every quinquennium for which we have data, at least one country reports an "extreme" value compared to others. The existence of outliers must therefore be taken into account in projecting the gap into future periods.

Figure 2 displays the narrowing of the gap in life expectancy between the sexes that has been reported in the literature for high-income countries - we use OECD membership as a distinguishing trait for illustrative purposes. Clearly, the trend is not present in non-OECD countries, though some, including Brazil and Uruguay, have experienced lower levels of the gap at higher levels of female life expectancy. Figure 2 also demonstrates that the trend in the gap is more easily distinguished when plotted against female life expectancy.

3 METHODOLOGY

Following the method first proposed by Carter & Lee (1992), the United Nations models female and male life expectancy independently, using a modification of the method to ensure that projections for the two sexes do not diverge (Li & Lee 2005). In fact, with the exception of the few studies designed specifically to forecast the gap in life expectancy between the sexes, all existing methods we reviewed project life expectancy for each sex independently (Soneji & King 2011, Shang et al. 2011, Cairns et al. 2011, Bongaarts 2009, Haberman & Renshaw 2008, Hyndman & Shahid Ullah 2007).

BHM is also designed as a one-sex model. To obtain joint probabilistic projections of female and male life expectancy, we need to model the relationship between the two. We opt to do this by projecting the gap in life expectancy by ordinary least squares regression using female life expectancy as a covariate. In this chapter, we provide a description of our model, as well as a justification for our choices.

Choice of covariates - Our objective is to obtain a model for projecting the gap in life expectancy that is comparable across all countries. Due to data limitations, we have a limited selection of covariates to include in our model, and are unable to incorporate any socioeconomic or behavioral data.

The narrowing mortality gap - As explained in the previous section, the pattern of decline in the gap in life expectancy has only been observed for high-income countries, and for some emerging economies. The question then arises: should we project that other countries will also follow this pattern? Vallin (2005) observed that "[t]here is no reason why the experience of English-speaking countries and Scandinavian countries [now extended to most high-income countries] should not become generalized, enabling men in most places to eventually regain some of the ground they lost during the 20th century." Bongaarts (2009) agrees that mortality patterns observed for high-income countries will most likely be followed by others: "[t]here is an expectation that, as developing countries evolve, they will also get rid of nonsenescent mortality, since non-senescent deaths unrelated to ageing (e.g., accidents, certain infections) can be avoided by effective public health and safety measures and through medical intervention."

We also draw the reader's attention to a distinguishing feature between OECD and non-OECD countries pictured in Figure 2: non-OECD countries have much lower female life expectancy than OECD countries, and it is only at high levels of female life expectancy that a narrowing in the sex gap has been observed. Thus, it is plausible that the reversal has not yet been seen in most non-OECD countries simply because they have not yet reached the female life expectancy level at which the gap begins to narrow.

Modeling the error term - Over the period 1950-2010, several countries experienced events that have had a significant impact on the gap between female and male life expectancy. Indeed, the gap in Bosnia and Herzegovina increased from 2.2 years to 5.3 years between 1950-1955 and 1985-1990, then jumped to 17.34 in 1990-1995 during a period of conflict before falling to 5.4 years in the next quinquennium. Similarly, the gap in Iraq increased from 6.33 years in 1980-1985 to 12.68 years in 1985-1990, and remained at approximately that level until the 1995-2000 quinquennium, when it dropped to 5.6 years. Because we can expect short-term shocks such as wars and epidemics to occur in the future, we must allow for outliers in our model.

Ours is not the first attempt to deal with outliers in mortality data: Hyndman & Shahid Ullah (2007) used robust statistics in developing their method for forecasting age-specific mortality and fertility rates observed over time. However, while their objective was to identify and remove data associated with extreme events, ours is to incorporate them in our projections so as to more realistically capture perturbations that may occur in the future.

In order to allow for outliers in our projections, we use the t-distribution rather than the normal distribution to model random perturbations, and estimate the parameters of the

t-distribution following the methodology developed by Lange et al. (1989) and Taylor & Verbyla (2004). Not allowing for negative gaps in life expectancy and not expecting future shocks to have a greater impact on the gap than the maximum impact observed between 1950-2010 (17.34 years), we restrict projections of the gap to the range of 0 to 18 years.

From projections of the gap to joint projections of life expectancy for the two sexes - As mentioned previously, in order to produce joint probabilistic projections of female and male life expectancy, we project the gap in life expectancy by ordinary least squares regression using BHM female life expectancy projections as a covariate. By subtracting our projection of the gap from the female life expectancy projection, we obtain the corresponding male life expectancy projection.

We choose to use female rather than male life expectancy as a basis for projecting the gap for several reasons: 1) the probabilistic model used by the UN to project fertility is based on women, so choosing females as a basis for this model would maintain consistency; 2) women live longer than men, so the window in which we are able to make projections is wider; and 3) in countries where mortality data is less complete or accurate, it is often better for women than for men. Although we believe these arguments to be sufficient to justify our choice, we have nevertheless attempted to build a model for the gap using BHM male life expectancy. As reported in the following section, this procedure yields inferior performance.

Mathematical specification - Our proposed model for the gap in life expectancy at birth between females and males represents the gap, $G_{c,t+1}$ for country c in the following quinquennium $t+1$ as a linear combination of:

- the gap in the current quinquennium $G_{c,t}$,
- female life expectancy at birth in the first observed quinquennium (1950-1955) $e_0f_{c,1953}$,
- female life expectancy at birth in the current quinquennium $e_0f_{c,t}$, and

- the number of years by which $e_0f_{c,t}$ exceeds 75, $e_0f_{c,t}^{>75}$.

To avoid making assumptions about future trends in the relationship between the gap and the level of female life expectancy, our model is divided in phases: the first applies when the level of female life expectancy falls within the window of observed data - that is, when it is lower than 86.17 years, the female life expectancy estimated for Japan in 2005-2010; the second applies when the level of female life expectancy exceeds 86.17. In summary, the following is the mathematical representation of our model:

$$G_{c,t+1} = \begin{cases} \beta_0 + \beta_1 e_0 f_{c,1953} + \beta_2 G_{c,t} + \beta_3 e_0 f_{c,t} + \beta_4 e_0 f_{c,t}^{>75} + \epsilon & lf_{ct} \leq \max(e_0 f_{ct}) \\ \gamma_1 G_{c,t} + \epsilon & lf_{ct} > \max(e_0 f_{ct}) \end{cases} \tag{1}$$

$$\epsilon \sim t(\mu = 0, \sigma^2 = 0.0665, \nu = 2)$$

Estimates of the parameters of this t distribution based on data from 158 countries over the period 1950-2010 contained in WPP 2008 are obtained by maximum likelihood using the R-package *hett* developed by Taylor (2009), and are reported in Table 1. While some heteroscedasticity was observed, we chose not to model it explicitly so as to keep the model as simple as possible. In addition, the original estimate of the degrees of freedom parameter was 2.07 (s.e. 0.13), which we rounded down to 2, again for simplicity.

4 MODEL VALIDATION

In order to assess the performance of our model, we calculate model parameters based on data from 1950 to 1995 only ($n = 1422$), and forecast the gap in life expectancy from 1995 to 2010. We then compare our results to WPP 2008 estimates of the gap for those three quinquennia. This process yields 474 out-of-sample projections.

We assess three attributes of our model: calibration, precision, and accuracy. An ideal model is perfectly calibrated (i.e. each x% prediction interval covers the true value x% of the time), precise (i.e. its prediction intervals are as narrow as possible), and accurate (i.e. makes the smallest errors in prediction). We measure accuracy by reporting the mean absolute error (MAE) of the median of the probabilistic projections.

We also compare the performance of our model to other potential gap projection methods. For the gap in life expectancy, we compare the performance of the gap model based on female life expectancy to the gap model based on the male life expectancy, as well as to the median difference between non-intersecting independent projections of female and male life expectancy under BHM ("BHM Projection"). We complete the study by also reporting the accuracy of a model that assumes that the gap would remain at 1990-1995 levels from 1995 to 2010 ("Constant 1995 Gap"). Our results are shown in Table 2.

4.1 Projections of the gap between female and male life expectancy at birth

We find that the model based on female life expectancy at birth has very good coverage and precision, and provides the most accurate projections. While the BHM Projection has very good coverage, it has low precision, particularly insofar as the 80% prediction interval is concerned, which is approximately twice as large as that of the other two models. It is also 14-23% less accurate than the other two models. The gap model based on male rather than female life expectancy has worse coverage and is less accurate than the female-based model.

4.2 Projections of male and female life expectancy at birth

Next, we use our projections of the gap and the BHM projections of female life expectancy to generate projections of male life expectancy, and we perform the same cross-validation exercise. We compare our results to three other sets of projections: 1) those of a UN analyst who computed life expectancy forecasts for 1995-2005 using one of the five prescribed UN models of gains in life expectancy at birth based on levels and trends over the period 1985-1995; 2) those that result from assuming that the gap observed in the period 1990-1995 would be constant for the following 15 years, and subtracting this value from the BHM female life expectancy trajectories; and 3) those that result from a direct application of BHM to males. These results are shown in the middle section of Table 2.

Once again, we find that the gap-based model has very good coverage and yields the most precise and most accurate estimates of male life expectancy at birth. While BHM has the best coverage, it has lower precision than the gap-based model, and lower accuracy than both the gap-based model and the model that assumes a constant gap in life expectancy. The UN projection has the worst results in terms of accuracy: using the BHM projection rather than the UN's improves accuracy by 38%, while the gap-based model improves it by 44%.

If we use our projections of the gap and the BHM projections of male life expectancy to generate projections of female life expectancy and perform the same cross-validation exercise, we find that the gap-based model does not perform better than BHM. Full results are reported in the bottom section of Table 2. This is partly because our projections of the gap are better calibrated, more precise, and more accurate when we use female e_0 as an input, and partly because BHM simply performs better for projecting female rather than male e_0. These findings serve to confirm our choice of deriving male life expectancy from projections of female life expectancy and of the gap rather than the reverse.

5 COUNTRY-SPECIFIC CASE STUDIES

In this chapter, we discuss our projections of the gap in life expectancy and male life expectancy for Ireland, Guatemala, Laos and Cambodia for the period 2010-2100. We chose these countries because they illustrate the four possible types of projection scenarios for the gap between female and male life expectancy: decline, rise followed by a decline, and continued rise. In Ireland, the narrowing in the gap has already been observed, and we project its continued narrowing; in Guatemala, the gap has not begun to narrow, but we project that it will in the near future; in Laos, we project a continued rise in the gap until 2030, followed by a decline; finally, in Cambodia, we project no narrowing in the gap until 2075.

5.1 Ireland

UN estimates of the gap in life expectancy between females and males in Ireland for the period 1950-2010, as reported in WPP 2008, are displayed in the the left panel of Figure 3, along with the WPP 2008 projection, our median projection, and our 80% and 95% prediction intervals (PI). These same measures for male life expectancy at birth are presented in the right panel. We note that the UN does not explicitly produce projections of the gap in life expectancy: both UN estimates and projections of the gap were calculated by subtracting estimates and projections of male life expectancy from estimates and projections of female life expectancy.

UN estimates show that Ireland experienced an increase in the gap between female and male life expectancy between the 1950-1955 and 1985-1990 quinquennia, from a gap of 2.5 years to a gap of 5.65 years. It then began a decline, reaching an estimated gap of 4.76 years in the 2005-2010 quinquennium. WPP 2008 projects the gap between female and male life expectancy to remain at 4.8 years for the entire period 2010-2050. In contrast, our model projects that the gap will continue to decline through 2100, reaching a level of 2.26 years in the 2045-2050 quinquennium with an 80%PI of (0.79, 4.04) and 1.94 years in the 2095-2100 quinquennium with an 80%PI of (0.47, 4.21). The UN projection of 4.8 years is higher than the upper bound of our 80%PI for the 2045-2050 quinquennium.

As far as male life expectancy at birth is concerned, the right panel of Figure 3 provides some interesting information. The median projections of the gap-based model are higher than the WPP 2008 projection. While WPP 2008 projects male life expectancy at birth in the 2045-2050 quinquennium to be 82.12 years, the median gap-based projection is 87.67 with a 80%PI of (85.20, 89.91). Again, the lower bound of the 80%PI is higher than the life expectancy projected by the UN. As we will discuss in further detail in Chapter 7 below, the fact that our projections of male life expectancy are higher than those of the UN implies that we project that Ireland will see a larger elderly population in the future and that the elderly population will be less dominated by females than currently expected.

5.2 Guatemala

Guatemala has not yet seen a narrowing of the gap between the life expectancy at birth of females and males. Indeed, the gap has increased from just 0.5 years in 1950-1955 to 7.11 in 1995-2000, and stagnated at approximately 7 years through 2010. As for Ireland, the UN projects that the gap will remain constant between 2010-2050 at a level of 7.1 years. However, our model suggests that after a brief rise to 7.07 years in the 2010-2015 quinquennium [80%PI of (6.55, 7.55)], the gap will begin to decrease, dropping to 4.65 years [80%PI of (2.76, 6.75)] in the 2045-2050 quinquennium, and to 2.05 years [80%PI of (0.43, 4.50)] in the 2095-2100 quinquennium.

The uncertainty in the BHM projection of life expectancy at birth of Guatemalan females being higher than that for Irish females, we obtain, as expected, projection intervals that are wider for Guatemala than for Ireland. Indeed, the width of the 80% PI in 2045-2050 for

Guatemala is 3.99 years, while it is 3.25 for Ireland. In the 2095-2100 quinquennium, it is 4.07 years for Guatemala and 3.74 years for Ireland.

Again, the gap-based median projections of male life expectancy at birth are higher than the WPP 2008 projection. While WPP 2008 projects a life expectancy of 74.52 in 2045-2050, the gap-based median projection is 77.70 [80%PI of (73.74, 82.19)].

5.3 Laos

In contrast to the previous two countries, estimates for Laos suggest that it has seen a decline in the gap between female and male life expectancy from 3.76 years in 1950-1955 to 2.50 years in 1975-1980, followed by a stabilization at approximately 2.5 years until 2000-2005, and an uptick to 2.8 years in the last observed period. WPP 2008 projects that the gap will increase to 4.33 years by the 2045-2050 period. In contrast, our median projection is that the gap will rise to 3.59 years in 2025-2030 [80%PI of (2.50 4.70)] and begin a decline in the 2035-2040 period, to 1.51 years in 2095-2100 [80%PI of (0.31, 3.96)]. Laos demonstrates that our model is capable of generating projections that rise and fall over time, following the evolution of female life expectancy in a country.

Once again, median projections of male life expectancy for Laos under the gap-based model are higher than the WPP 2008 projection, with WPP 2008 projecting a life expectancy of 73.65 years in 2045-2050, while the gap-based projection is 79.72 [80%PI of (72.60, 87.30)]. The width of the prediction intervals for male life expectancy in Laos are visibly much larger than for Guatemala or Ireland, which is consistent with the higher uncertainty involved in projecting these demographic quantities in Laos.

5.4 Cambodia

Our final example is Cambodia - a country with a seemingly erratic pattern of the gap in life expectancy between females and males, which has generally been on an upwards trajectory since the 1950-1955 quinquennium, and which has most recently been estimated at 3.60 years.

In this case, the WPP 2008 projection for the gap and our model's median projections agree quite well, even regarding the pace of the increase in the gap. In the 2045-2050 quinquennium, WPP 2008 projects a gap of 4.60 years, while our model's median projection is 4.61 years, with an 80%PI of (2.97, 6.21). We project that this gap will rise to 5.01 years [80% PI of (3.06, 6.86)] in the 2070-2075 quinquennium and remain at that level for approximately 15 years before beginning to drop. Our median projection for the 2095-2100 quinquennium is 4.75 years [80% PI of (2.07, 7.11)]. Since the level of female life expectancy in Cambodia is low (62.61 years in 2005-2010), our model projects a growth in the gap, following the trend of other nations. Thus, Cambodia shows that our model can - and does - project increasing trajectories for the gap.

Cambodia also offers an example of a situation where the gap-based median projections of male life expectancy are lower than the WPP 2008 projection. While WPP 2008 projects male life expectancy in Cambodia to be 72.14 years in 2045-2050, the median gap-based projection is 65.05 [80% PI of (59.66, 69.39)].

6 DISCUSSION

In this paper, we have described a methodology for obtaining joint probabilistic projections of female and male life expectancy. This methodology is built on previous work by Chunn et al. (2010) which uses a Bayesian hierarchical model framework to enhance the flexibility of the double-logistic function of gains in life expectancy employed by the United Nations and produces projections of life expectancy for each sex independently. Given probabilistic projections of life expectancy for females, we model the gap in life expectancy between females and males, and obtain corresponding projections of male life expectancy by combining these two quantities.

Because our objective is to project the gap in life expectancy between the sexes for *all* countries in the world, in contrast to previous studies, our model must 1) be based only on available data, 2) be comparable across countries, and 3) account for possible shocks such as armed conflicts, as observed in the past. Thus, we model the gap in a future period for a particular country simply as a function of the gap and female life expectancy in the current period in that country and a t-distributed random perturbation based on the experience of all countries. Having observed that high-income countries that have reached high levels of female life expectancy have also experienced a narrowing in the gap between the sexes, we project that developing countries will follow the trend. In addition, acknowledging that there is no current expectation that males will live longer than females in the future and that there is no reason to expect larger differentials than observed in the past, we allow projected gaps to vary only between 0 and 18 years.

We estimate all the parameters of our model based on life expectancy estimates reported in WPP 2008. We evaluate it using out-of-sample projections for the period 1995-2010, and compare our results to current UN methods as well as to probabilistic projections of the gap based on independent female and male projections of life expectancy. We demonstrate that our model performs better than several independent projections of male and female life expectancy, insofar as coverage, precision and accuracy are concerned.

Despite its good performance, we recognize that our model is subject to several limitations, the first of which is our choice of covariates. Often, life expectancy is projected based on expectations regarding the impact of cause-specific mortality rates (Bongaarts 2009). Indeed, sex differences in mortality could be explained by a multitude of factors, including differences in the distribution of social and biological protective and risk factors by sex, such as socioeconomic status, social relationships, health behaviors, and biological indicators of health (see Rogers et al. 2010 for a review of these factors). In addition, it has been shown not only that the narrowing sex differences in mortality in high-income nations could be explained by smoking patterns between the sexes (Pampel 2005; Preston & Wang 2006), but also that differences in the pattern of the decline in various countries could be explained in part by the pattern of cigarette consumption (Staetsky 2009; Bobak 2003).

Given these findings, it is reasonable to suggest that projections of the gap in life expectancy should be based on differences in expectations of improvements in cause-specific mortality rates for one sex relative to the other, or on smoking patterns, as in previous studies in high-income countries (Pampel 2005; Preston & Wang 2006). However, doing so would require projecting many socioeconomic quantities that affect a population's health, while facing the aforementioned lack of reliable data even for basic demographic quantities in many countries. While this is impossible for the moment, it is a potential source of improvement in the future. Nevertheless, as Booth (2006) warns, one must be cautious to remain focused on the demographic rather than the socioeconomic forecasting problem, as it may be no less difficult to forecast the socioeconomic determinants (even given good data) than to directly forecast the demographic variable (Keyfitz 1982).

In addition, it would be interesting to study the impact of the joint probabilistic projection of male and female life expectancy on overall population projections. Our work leads us to expect males to live longer than projected in WPP 2008; consequently, we would expect this to result in larger numbers of older individuals, and a more balanced sex balance amongst the old. An examination of the potential support ratio (or PSR, defined as the ratio of the number of individuals between the ages of 15 and 64 to the number of individuals 65 and older), the proportion of oldest old in the population (defined as the ratio of the number of individuals over 85 years of age to the total population), and the proportion of females to males among the oldest old using our projections of life expectancy may yield interesting observations regarding the composition of elderly populations in the future.

Finally, it will be interesting to monitor the evolution of the gap between female and male life expectancy in developing countries to determine whether they will truly follow the lead of high-income countries, as many experts expect. As new estimates and projections of life expectancy are published, the model we propose could be applied and compared to existing projection methods to further validate its performance. A good first step would be to repeat the experiment on data and projections contained in WPP 2010, published last May. Given that WPP 2010 includes probabilistic projections of the total fertility rates, comparing results on full population estimates would in itself be an interesting exercise.

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Tables

Table 1: Estimates and Standard Errors of Parameters used in Gap Projection Model

Parameter	Estimate	Standard Error
β_0	-0.158	0.059
β_1	0.006	0.001
β_2	0.954	0.004
β_3	0.003	0.001
β_4	-0.094	0.006
γ_1	0.969	0.004

Table 2: Results for 15-year out-of-sample cross-validation for four different projection models for 1) the gap between female and male life expectancy at birth, 2) male life expectancy at birth, and 3) female life expectancy at birth.

	Calibration and Precision				Accuracy
	Coverage		Avg. Half-Width		
	80%	95%	80%	95%	MAE
GAP IN $\mathbf{e_0}$					
UN Projection	–	–	–	–	0.78
Constant 1995 Gap	–	–	–	–	0.73
BHM Projection	0.78	0.92	1.48	2.22	0.80
Male e_0-based Model	0.68	0.93	0.75	1.57	0.70
Female e_0-based Model	0.74	0.94	0.75	1.56	0.65
MALE $\mathbf{e_0}$					
UN Projection	–	–	–	–	2.32
Constant 1995 Gap	0.64	0.85	1.61	2.46	1.42
BHM Projection	0.79	0.94	2.05	3.13	1.44
Gap-based Model	0.76	0.92	1.88	2.98	1.31
FEMALE $\mathbf{e_0}$					
UN Projection	–	–	–	–	1.94
Constant 1995 Gap	0.86	0.96	2.05	3.14	1.14
BHM Projection	0.75	0.90	1.61	2.46	1.16
Gap-based Model	0.89	0.97	2.23	3.48	1.17

Figures

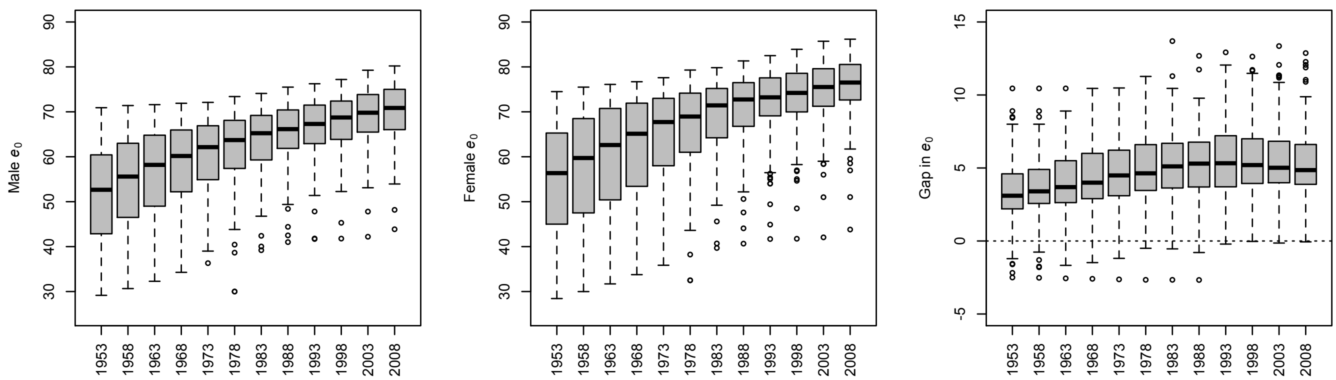

Figure 1: Trends in male and female life expectancy at birth (e_0) and the gap between them for all 158 countries under study over the period 1950-2010, based on estimates in WPP 2008. The dark lines in the boxplots indicate median values; the gray boxes extend to the first and third quartile, and the whiskers extend to the most extreme data point which is no more than 1.5 times the interquartile range from the box. Note that each quinquennium is identified by its middle year (e.g. 1950-1955 →1953).

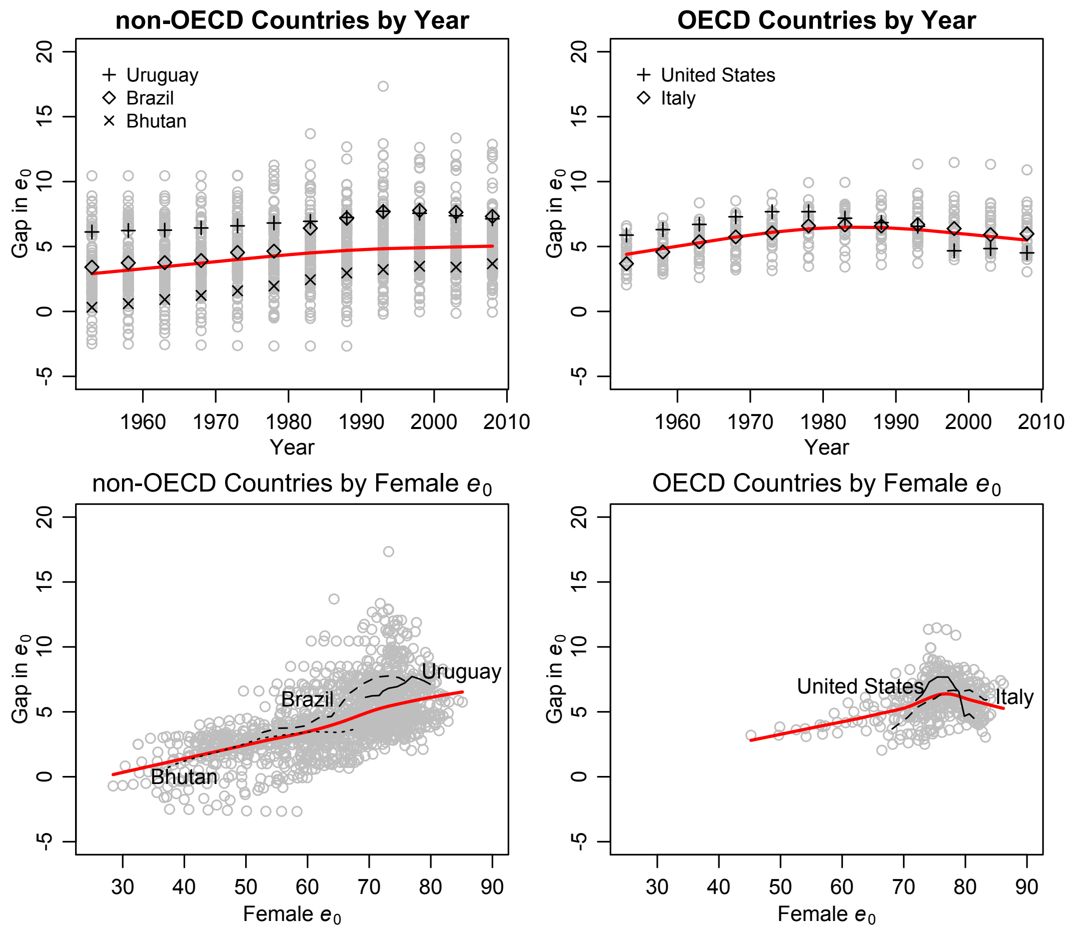

Figure 2: Trends in the gap between female and male life expectancy at birth for OECD (right panels) and non-OECD countries (left panels) over time (top panels) and over female life expectancy at birth (bottom panels) for the period 1950-2010 as reported in WPP 2008. The trend in the gap for five countries is independently reported for illustrative purposes: Bhutan, Brazil, Italiy, United States and Uruguay.

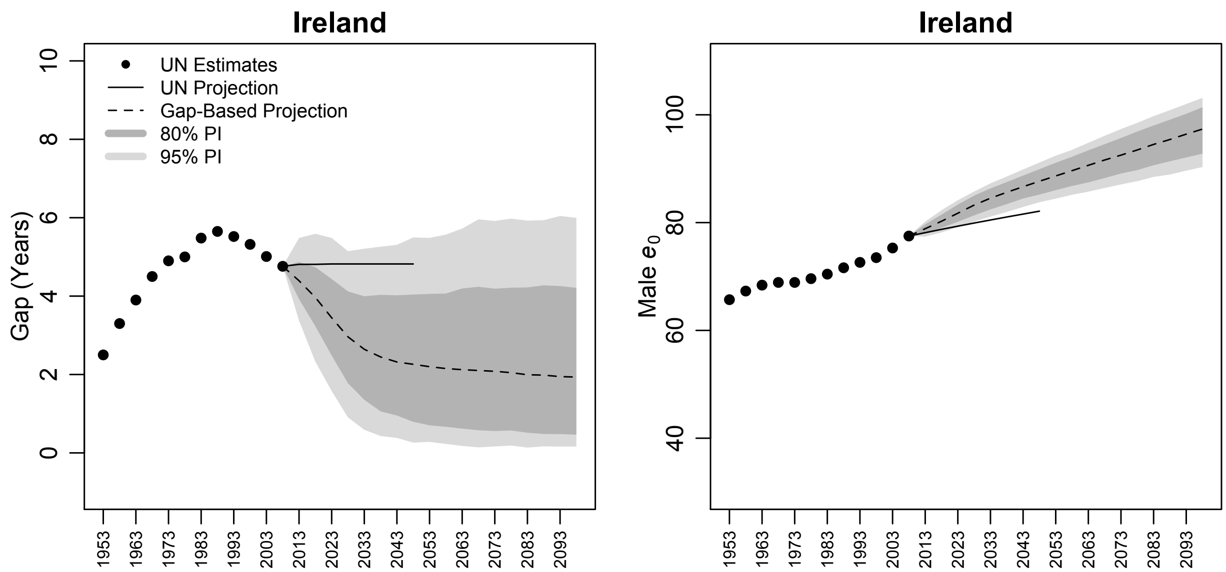

Figure 3: Estimates and projections of the gap in life expectancy between females and males (left panel) and of male life expectancy (right panel) for Ireland. WPP 2008 estimates are shown as black circles; WPP 2008 projections to 2050 are shown as a black line. The median projection using our gap-based model is showin as a dotted line, along with its 80% and 95% PI.

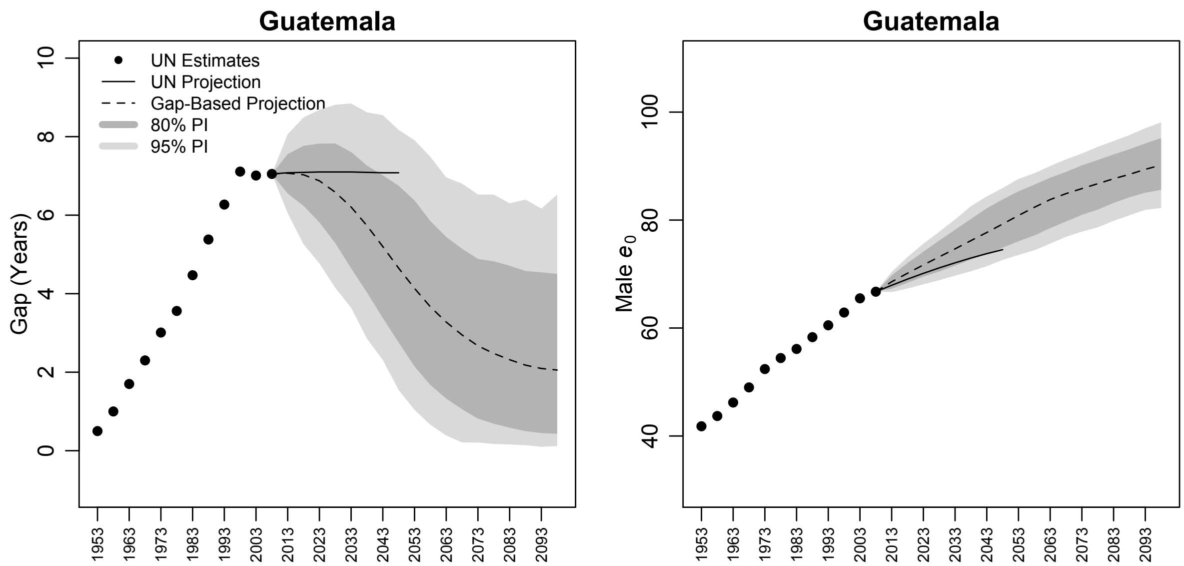

Figure 4: Estimates and projections of the gap in life expectancy between females and males (left panel) and of male life expectancy (right panel) for Guatemala. WPP 2008 estimates are shown as black circles; WPP 2008 projections to 2050 are shown as a black line. The median projection using our gap-based model is showin as a dotted line, along with its 80% and 95% PI.

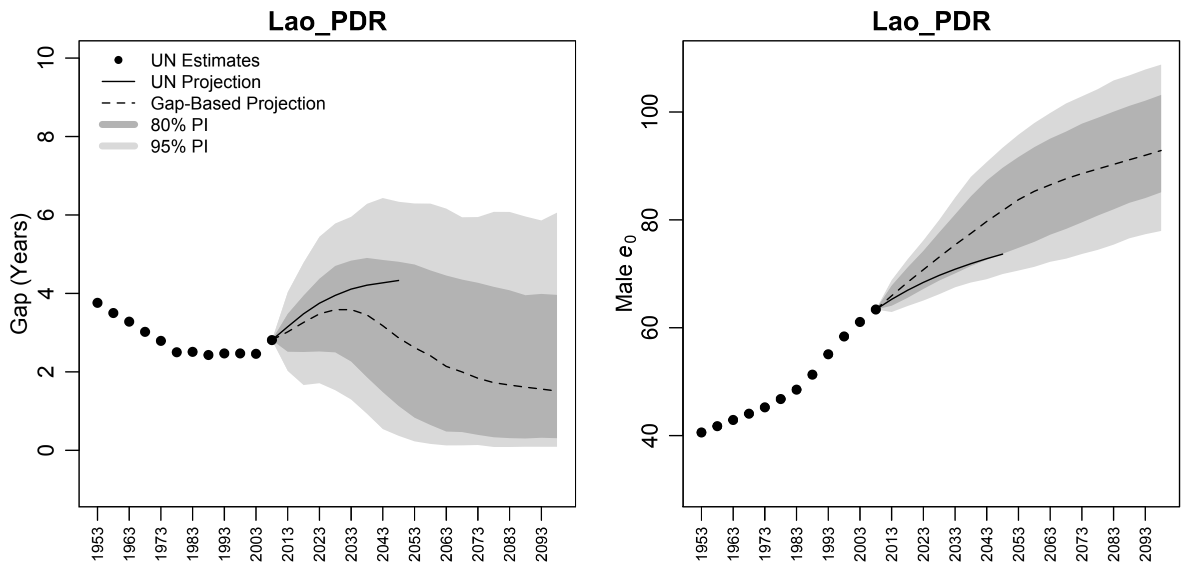

Figure 5: Estimates and projections of the gap in life expectancy between females and males (left panel) and of male life expectancy (right panel) for Laos. WPP 2008 estimates are shown as black circles; WPP 2008 projections to 2050 are shown as a black line. The median projection using our gap-based model is showin as a dotted line, along with its 80% and 95% PI.

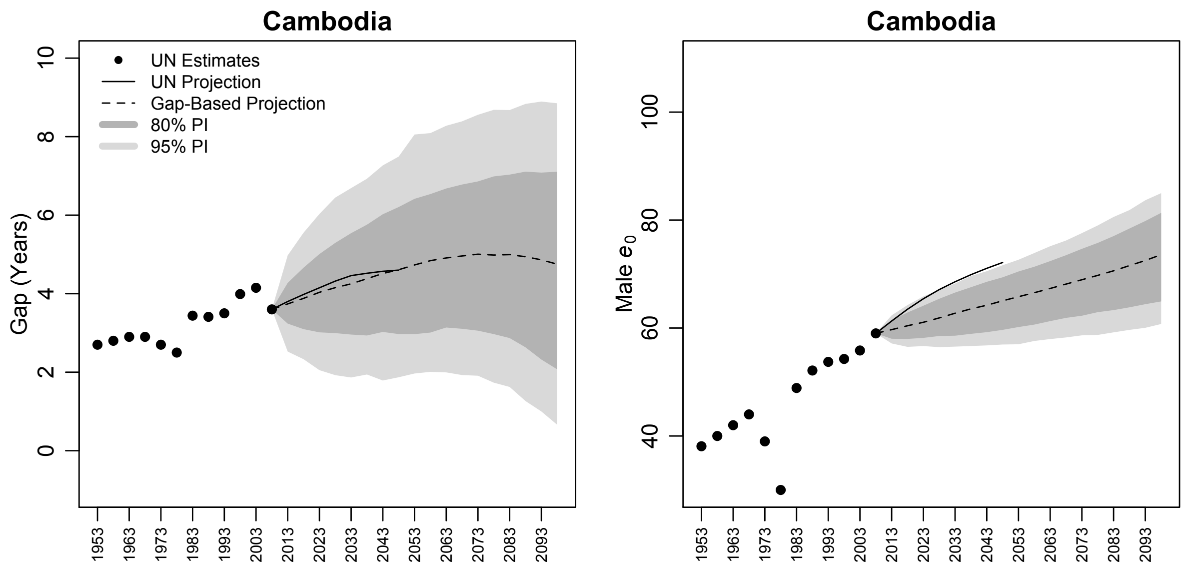

Figure 6: Estimates and projections of the gap in life expectancy between females and males (left panel) and of male life expectancy (right panel) for Cambodia. WPP 2008 estimates are shown as black circles; WPP 2008 projections to 2050 are shown as a black line. The median projection using our gap-based model is showin as a dotted line, along with its 80% and 95% PI.

Acknowledgments

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